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ESTIMATION OF THE OPERATING CHARACTERISTICS OF ITEM RESPONSE CATEGORIES II: FURTHER DEVELOPMENT OF THE TWO-PARAMETER BETA METHOD

FUMIKO SAMEJIMA

DEPARTMENT OF PSYCHOLOGY
UNIVERSITY OF TENNESSEE
KNOXVILLE, TENN. 37916

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The Two-Parameter Beta Method, was introduced in the previous study as a method of estimating the operating characteristics of a test item, and it has proved to be as efficient as the Normal Approximation Method, for a set of a simulated data of 500 hypothetical examinees having a uniform latent trait distribution between -2.475 and 2.475. Both methods are characterized by: (1) the use of a relatively small number of subjects, like 500, in the whole procedure of estimation, (2) without assuming any prior mathematical model, and (3) the use of the estimated joint distribution of the latent trait and its maximum likeli- (over)

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hood estimate. In the Two-Parameter Beta Method, the method of moments is adopted to approximate the probability density function of the maximum likelihood estimate, using polynomials of degree 3 and 4. The first two conditional moments of the latent trait, given the maximum likelihood estimate, are derived from theory and computed for the data for each value of the maximum likelihood estimate. The conditional distribution of the latent trait, given the maximum likelihood estimate, is approximated by a Beta distribution using the method of moments, with two a priori set parameters and two estimated parameters from the conditional moments. Five scores of the latent trait are calibrated according to this approximated Beta distribution. For each of the ten binary items, whose item characteristic functions are to be estimated, the set of frequency ratios of these scores for the group of subjects who have answered correctly to the total subjects for each of the 20 intervals of the latent trait is taken as the estimated item characteristic function.

In the present study, the Two-Parameter Beta Method is further developed to add two more variations, and to compare the result with the one obtained in the previous study, In the first method, the resulting set of calibrated latent trait scores for each of the two groups, i.e., the group of subjects who have answered correctly to the binary item and those who answered incorrectly, is approximated by a polynomial of degree 3 or 4 by the method of moments. The ratio of the polynomial thus obtained for the correct group to the sum of these two polynomials is the estimated item characteristic function of this binary item. In the second method, instead of calibrating five latent trait scores for each maximum likelihood estimate, the whole conditional probability density function of the latent trait, given the maximum likelihood estimate, is used. These density functions are summed up for each of the correct and the total groups, and the ratio of the summed density functions for the correct group to those for the total group is the estimated item characteristic function of the binary item. This is done for the case in which a polynomial of degree 3 is used to approximate the set of maximum likelihood estimates of the 500 subjects, and also for the case in which a polynomial of degree 4 is used, and they are called Degree 3 Case and Degree 4 Case respectively.

For the purpose of evaluating these results, the theoretical bivariate density function of the latent trait and its maximum likelihood estimate is derived from the uniform distribution of the latent trait and the conditional distribution of the maximum likelihood estimate, which is approximated by the normal distribution with the latent trait, θ, and the inverse of the test information function, $I(\theta)$, as the two parameters. As in the previous study, the test information function assumes the constant value, 21.63, for the interval $(-2.5, 2.5)$ of θ. From this bivariate density function, the conditional density function of the latent trait, given the maximum likelihood estimate, is obtained for each of the 500 subjects. These conditional density functions are summed up for the correct group and the total group for each of the ten binary items, and the ratio of the former to the latter is used as the criterion operating characteristic, in addition to the frequency ratio of the correct group to the total group for each of the 20 intervals of θ with the width of 0.25.

It turned out that 1) the result of the second method is almost identical with the criterion operating characteristic in both Degree 3 Case and Degree 4 Case, and far better than the frequency ratios; 2) the result of the first method proves that the approximation of the sets of calibrated latent trait scores, $\tilde{\theta}$, by the polynomials of degree 3 or 4 improves the closeness of the curve to the theoretical item characteristic function, to some extent, but not too much.

The least square principle is used in estimating two parameters following the normal ogive model, for each result.

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ABSTRACT

The Two-Parameter Beta Method, was introduced in the previous study as a method of estimating the operating characteristics of a test item, ~~and it~~ has proved to be as efficient as the Normal Approximation Method, for a set of simulated data of 500 hypothetical examinees having a uniform latent trait distribution between -2.475 and 2.475 . Both methods are characterized by: (1) the use of a relatively small number of subjects--like 500--in the whole procedure of estimation; (2) without assuming any prior mathematical model; and (3) the use of the estimated joint distribution of the latent trait and its maximum likelihood estimate. In the Two-Parameter Beta Method, the method of moments is adopted to approximate the probability density function of the maximum likelihood estimate, using polynomials of degree 3 and 4 . The first two conditional moments of the latent trait, given the maximum likelihood estimate, are derived from theory and computed for the data for each value of the maximum likelihood estimate. The conditional distribution of the latent trait, given the maximum likelihood estimate, is approximated by a Beta distribution using the method of moments, with two a priori set parameters and two estimated parameters from the conditional moments. Five scores of the latent trait are calibrated according to this approximated Beta distribution. For each of the ten binary items, whose item characteristic functions are to be estimated, the set of frequency ratios of these scores for the group of subjects who have answered correctly to the total number of subjects for each of the 20 intervals of the latent trait is taken as the estimated item characteristic function.

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The research was conducted at the principal investigator's laboratory, 409 Austin Peay Hall, Department of Psychology, University of Tennessee, Knoxville, Tennessee. The computer programming work was greatly assisted by one of her graduate assistants, Yeh Ching-Chuan. The other assistants working for her at various times include Merle C. Steelman, Robert L. Trestman, C.I. Bonnie Chen, Li-Jen Jenny Chen, and an undergraduate student, Philip S. Livingston.

TABLE OF CONTENTS

I Introduction

The Two-Parameter Beta Method has been introduced as a method of estimating the operating characteristics of item response categories (Samejima, 1977d). Just like the Normal Approximation Method, its main features will be summarized as follows.

(1) A relatively small number of subjects, e.g., 500, are needed in the whole process of estimation.

(2) No prior mathematical models are assumed for the operating characteristics.

Throughout the process, the method of moments (Elderton and Johnson, 1969, Johnson and Kotz, 1970) is applied effectively, and the test *information function and its asymptotic properties* in connection with the maximum likelihood estimate (Samejima, 1977a, 1977b, 1977c) are fully utilized. On the other hand, there are some restrictions in this method, i.e., 1) a set of items, whose operating characteristics are known, is needed, in addition to a new set of items whose operating characteristics are to be estimated; and 2) the old set of items should provide us with a constant test information function throughout the range of ability, or latent trait, of our interest. This second restriction is of no serious obstacle if we use tailored testing data with an a priori set test information function as the criterion for terminating the presentation of new items (Samejima, 1977a, in press a, in press b), although it is more difficult to realize in a paper-and-pencil testing situation. A modification of this procedure to eliminate the first restriction is under consideration, in order to cover the

situation where no such old set of items is available. In this paper, however, the second restriction still holds.

The main objective of this study is to produce some variations of the Two-Parameter Beta Method without changing the empirical setting. For this purpose, the same simulated data as were used in the previous studies (Samejima, 1977b, 1977d) are used throughout the study, with 500 hypothetical examinees whose ability distributes uniformly for the range of ability [-2.5, 2.5]. Thus the constant value of the test information function of the old set of items is 21.63, which provides 0.215 for the standard deviation of the conditional distribution of the maximum likelihood estimate, given ability θ, and is large enough to approximate the conditional distribution of the maximum likelihood estimate $\hat{\theta}$ by $N(\theta, 0.215^2)$. The new items, whose operating characteristics are to be found, are the same ten binary items which follow the normal ogive model.

The present research is divided into two parts, i.e., the estimation of the operating characteristics 1) by fitting a polynomial to each set of $\tilde{\theta}$, which are calibrated by the Monte Carlo method (Samejima, 1977d) and are classified in terms of the item score of each of the ten binary items, by the method of moments, and taking its ratio to the sum of all the polynomials; and 2) by totalling the whole conditional probability densities of θ, given its maximum likelihood estimate, each of which is approximated by a Beta density function with two a priori set parameters and two other estimated parameters from the first two conditional moments of θ, in terms of each item score category of a new item, and taking its ratio to the total sum of the conditional

density functions. For convenience, hereafter the former method will be called the <u>Curve Fitting Method for</u> $\tilde{\theta}$ in contrast to the <u>Histogram Ratio Method for</u> $\tilde{\theta}$ which indicates the method introduced in the preceding research report (Samejima, 1977d), whereas the latter method will be called the <u>Conditional P.D.F. Method</u>, within the context of the Two-Parameter Beta Method.

II <u>Curve Fitting Method for</u> $\hat{\theta}$

The simulated data used for this part of the research are exactly the same as those adopted in the previous study of the Two-Parameter Beta Method, up to the calibration of 2500 values of $\hat{\theta}$. To summarize:

(1) The set of 500 maximum likelihood estimates, $\hat{\theta}$, were graduated by a polynomial by the method of moments. In so doing, both polynomials of degrees 3 and 4 were used. The resulting polynomials are:

(2.1) $0.22416 - 0.00351\hat{\theta} - 0.01873\hat{\theta}^2 + 0.00095\hat{\theta}^3$

and

(2.2) $0.19620 + 0.00238\hat{\theta} + 0.01319\hat{\theta}^2 - 0.00062\hat{\theta}^3 - 0.00427\hat{\theta}^4$.

For convenience, whenever the first polynomial is used for the probability density function of $\hat{\theta}$, we call it Degree 3 Case, and it is called Degree 4 Case if the second polynomial is used.

(2) The conditional mean and variance of θ , given $\hat{\theta}$, are given by

(2.3) $$E(\theta|\hat{\theta}) = \hat{\theta} + \sigma^2 \frac{d}{d\hat{\theta}} \log g(\hat{\theta})$$

and

(2.4) $$\text{Var.}\ (\theta|\hat{\theta}) = \sigma^4 \frac{d^2}{d\hat{\theta}^2} \log g(\hat{\theta}) + \sigma^2 ,$$

where $\sigma = 0.215$ and $g(\hat{\theta})$ is the probability density function of $\hat{\theta}$ approximated by one of the polynomials given in (1).

(3) The conditional distribution of θ , given the maximum likelihood estimate $\hat{\theta}$, is assumed to be a Beta distribution, whose probability density function is given by

$$(2.5)\qquad \hat{\phi}(\theta|\hat{\theta}) = [B(p_{\hat{\theta}}, q_{\hat{\theta}})]^{-1}(\theta - a_{\hat{\theta}})^{p_{\hat{\theta}}-1}(b_{\hat{\theta}} - \theta)^{q_{\hat{\theta}}-1} \cdot (b_{\hat{\theta}} - a_{\hat{\theta}})^{-(p_{\hat{\theta}}+q_{\hat{\theta}}-1)} ,$$

where $a_{\hat{\theta}}$ and $b_{\hat{\theta}}$ are $\hat{\theta} - 2.55\sigma^2 = \hat{\theta} - 0.54825$ and $\hat{\theta} + 2.55\sigma^2 = \hat{\theta} + 0.54825$ respectively, $p_{\hat{\theta}}$ and $q_{\hat{\theta}}$ are obtained from the conditional moments by

$$(2.6)\qquad p_{\hat{\theta}} = M_1^2(1 - M_1)M_2^{-1} - M_1$$

and

$$(2.7)\qquad q_{\hat{\theta}} = M_1(1 - M_1)^2M_2^{-1} - (1 - M_1) ,$$

where

$$(2.8)\qquad M_1 = [E(\theta|\hat{\theta}) - a_{\hat{\theta}}]\ (b_{\hat{\theta}} - a_{\hat{\theta}})^{-1}$$

and

$$(2.9)\qquad M_2 = E[\{\theta - E(\theta|\hat{\theta})\}^2\ |\hat{\theta}]\ (b_{\hat{\theta}} - a_{\hat{\theta}})^{-2} ,$$

respectively, and $B(p_{\hat{\theta}}, q_{\hat{\theta}})$ is the Beta function with $p_{\hat{\theta}}$ and $q_{\hat{\theta}}$ as its two parameters.

(4) Following this approximated conditional distribution of θ, given $\hat{\theta}$, five scores were calibrated for each maximum likelihood estimate by the Monte Carlo method, and these scores are symbolized by $\tilde{\theta}$. This procedure provided us with 2500 $\tilde{\theta}$'s in total in Degree 3 Case. In Degree 4 Case, however, this number is only 2465, since we had to discard seven hypothetical

examinees, or seven extreme values of the maximum likelihood estimate, either because of their negative values of the estimated $g(\hat{\theta})$ or of their negative values of the estimated conditional variance.

After this step, instead of taking the ratio of the frequency distribution of $\tilde{\theta}$ for the hypothetical subjects, who gave the correct answer to a new item g, to the frequency distribution of the total set of $\tilde{\theta}$, as we did in the Histogram Ratio Method, a polynomial of degree 3 or 4 is fitted by the method of moments, to the set of $\tilde{\theta}$ for the hypothetical examinees who answered item g correctly as well as to the set of $\tilde{\theta}$ for those who answered item g incorrectly. Then the two polynomials are added together to make a new polynomial for the total group of examinees, and the estimated item characteristic function is obtained as the ratio of the polynomial for the "success" group of examinees to the polynomial for the total group. In so doing, again, both polynomials of degrees 3 and 4 were used. For convenience, hereafter, we shall call these four cases Degree 3-3 Case, Degree 3-4 Case, Degree 4-3 Case and Degree 4-4 Case, respectively, with the first number indicating the degree of the polynomial used for $g(\hat{\theta})$, whereas the second number indicates the degree of the polynomial used in graduating the set of $\tilde{\theta}$ for each of the success and failure groups.

The simple unweighted least square method is used to evaluate the result, by obtaining the estimates of the two parameters in the normal ogive model, as was done previously (Samejima, 1977b).

The model specifies the item characteristic function, $P_g(\theta)$, for item g in the form

$$(2.10) \qquad P_g(\theta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a_g(\theta - b_g)} e^{-\frac{u^2}{2}} du ,$$

where a_g is the discrimination parameter and b_g is the difficulty parameter. Let $\tilde{P}_g(\theta_j)$ be the estimated value of the item characteristic function for the midpoint of the j-th interval, and define ζ_{gj} such that

$$(2.11) \qquad \tilde{P}_g(\theta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\zeta_{gj}} e^{-\frac{u^2}{2}} du .$$

Thus following the least square principle, we define Q by

$$(2.12) \qquad 2Q = \sum_{j=1}^{m} (\zeta_{gj} - a_g(\theta_j - b_g))^2 ,$$

and differentiating Q with respect to a_g and b_g and setting the results equal to zero, we obtain

$$(2.13) \qquad \frac{\partial Q}{\partial a_g} = \sum_{j=1}^{m} (\zeta_{gj} - a_g(\theta_j - b_g))(-\theta_j + b_g) = 0$$

and

$$(2.14) \qquad \frac{\partial Q}{\partial b_g} = \sum_{j=1}^{m} (\zeta_{gj} - a_g(\theta_j - b_g))\, a_g = 0 .$$

The above equations lead us to the estimates of a_g and b_g such that

$$(2.15) \qquad \hat{a}_g = \text{Cov.}\ (\zeta_{gj}, \theta_j)(\text{Var.}\ (\theta_j))^{-1}$$

and

$$(2.16) \qquad \hat{b}_g = \bar{\theta} - (\text{Cov. } (\zeta_{gj}, \theta_j))^{-1} \text{Var. } (\theta_j) \bar{\zeta}_g \text{ ,}$$

where $\bar{\theta}$ and $\bar{\zeta}_g$ are the means of θ_j and ζ_{gj} respectively. These estimates, $\hat{a}_g$ and $\hat{b}_g$, are to be compared with their respective parameters, a_g and b_g.

Throughout the present study, the interval width used for the parameter estimation is 0.2 .

III Results

Figures 3-1 and 3-2 present the histograms of the frequency distribution of $\tilde{\theta}$ for both the success and failure groups of examinees for each of the ten binary items, and the approximated polynomials of degree 3 and 4

$$\alpha + \beta\theta + \gamma\theta^2 + \delta\theta^3 \tag{3.1}$$

and

$$\alpha + \beta\theta + \gamma\theta^2 + \delta\theta^3 + \nu\theta^4 \ , \tag{3.2}$$

which were obtained by the method of moments described in the preceding section.

As is expected, the polynomials of degree 4 show a slightly better fit to the histograms than the polynomials of degree 3, but overall the fit is fairly good for both types of curves. Another conspicuous feature of these curves is that outside the ranges of θ where the histograms are located there exist, for many items, rapid increases and decreases. This warns us that we should be careful to avoid using these meaningless parts of the polynomials in the estimation of item characteristic functions. (Note that in these graphs the two histograms for the success and failure groups are drawn in such a way that the sum total of the two histograms makes the relative frequency distribution of $\tilde{\theta}$. In other words, these histograms are proportioned according to the relative frequencies of those who answered correctly to the item and those who did not, to make the sum of these areas unity.)

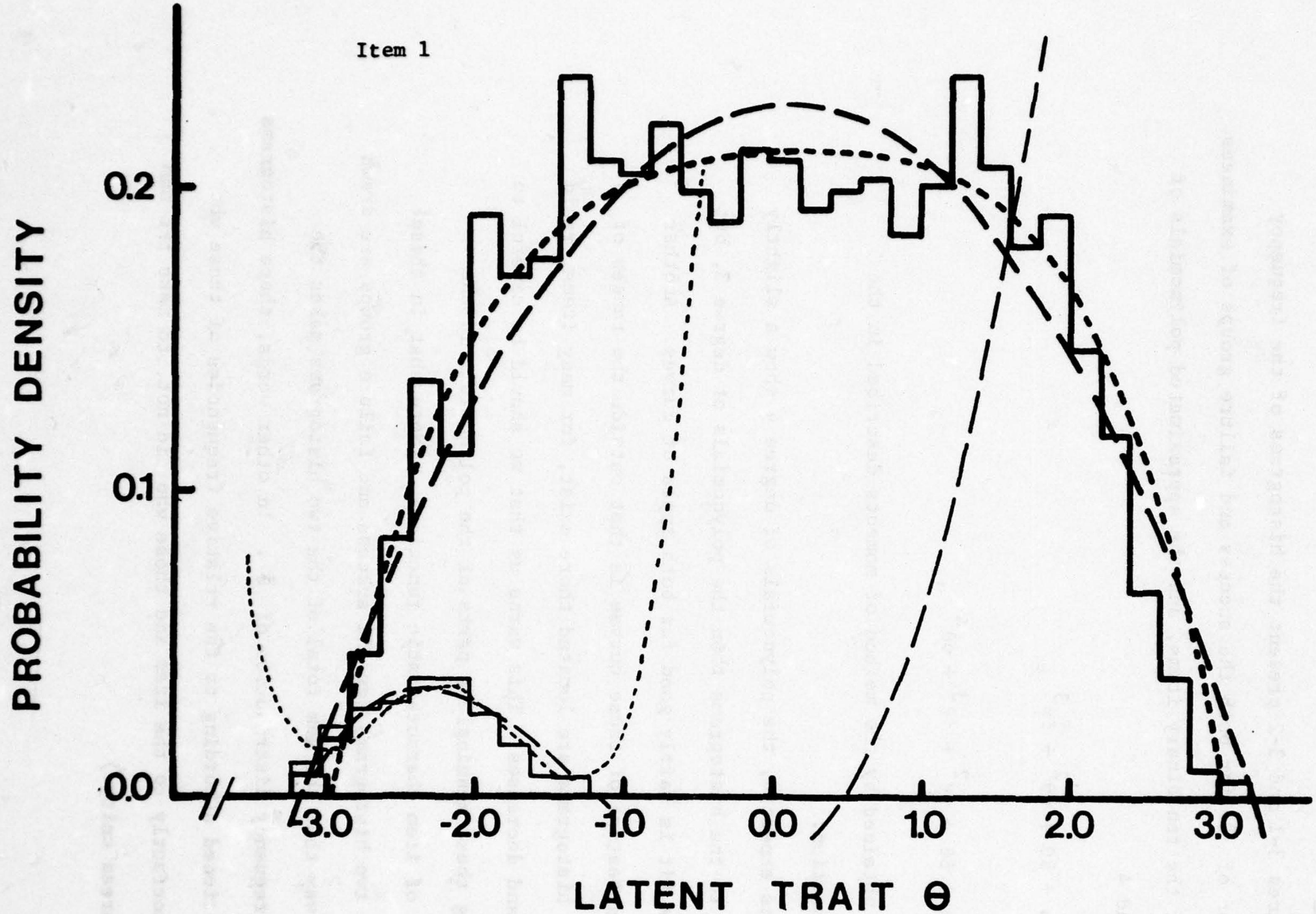
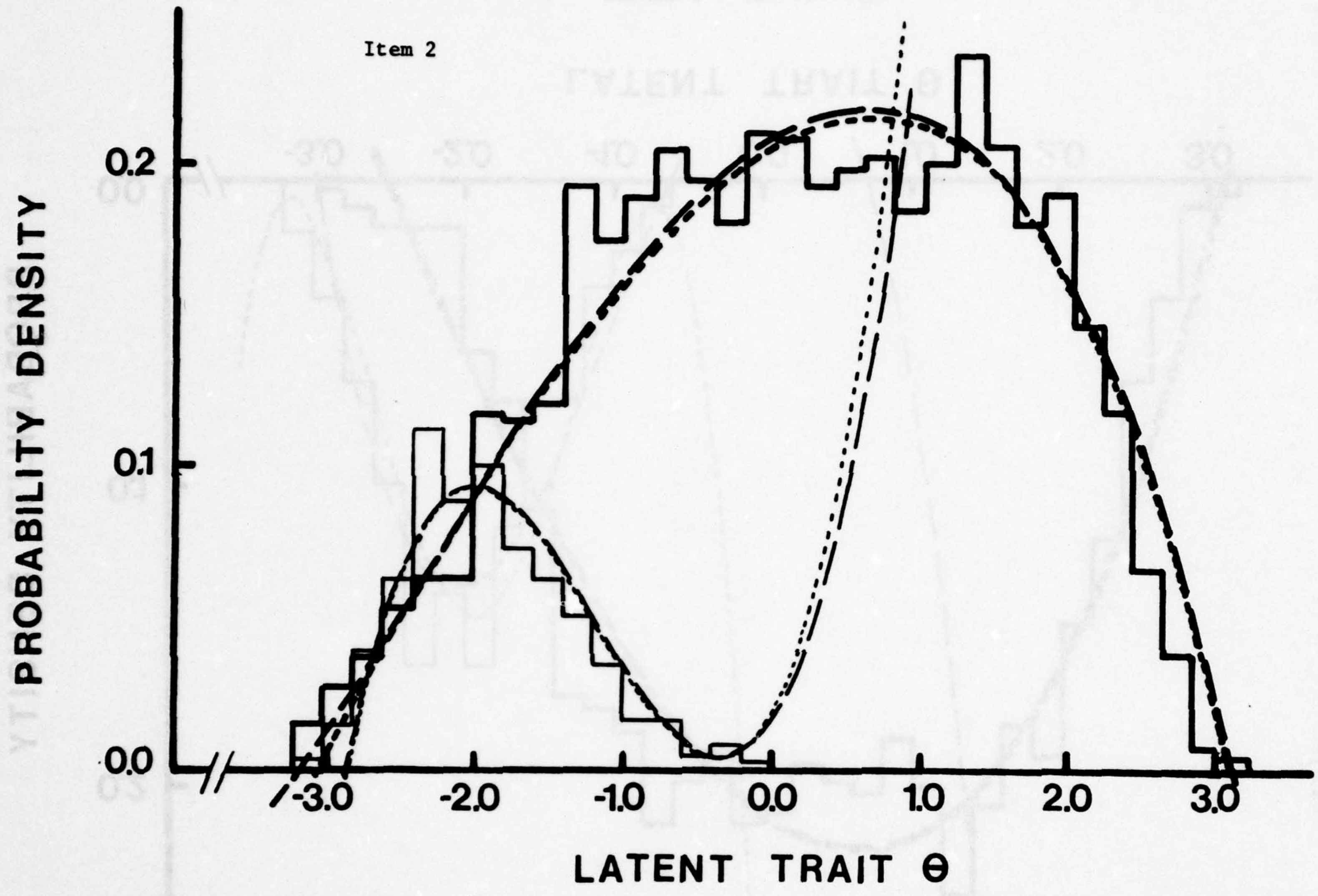


FIGURE 3-1

Frequency Ratios of the Success (Thick Line) and the Failure (Thin Line) Groups and the Corresponding Polynomials of Degree 3 (Broken Curve) and of Degree 4 (Dashed Curve) for Each of the Ten Binary Items Following the Normal Ogive Model: Degree 3-3 and 3-4 Cases



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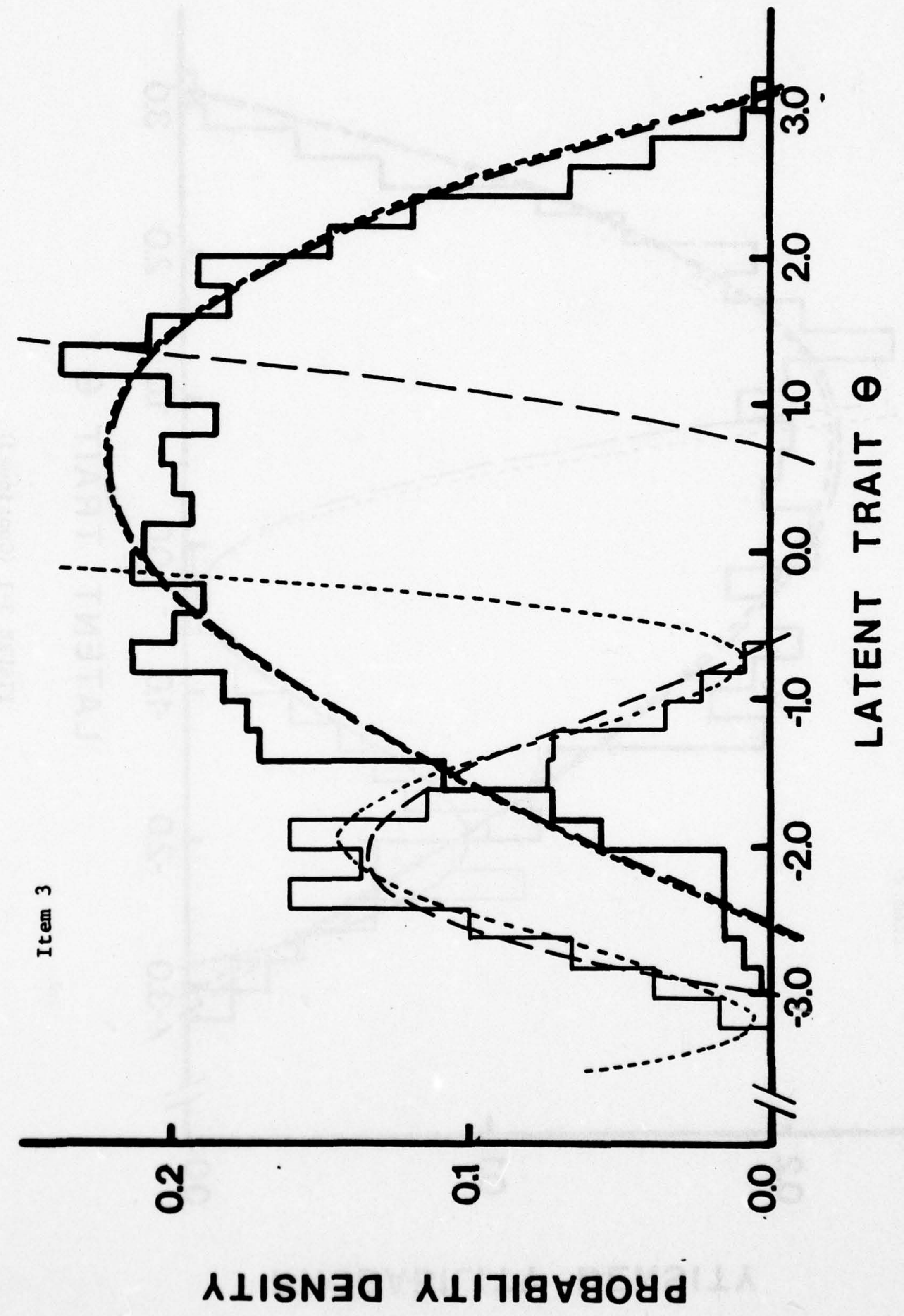

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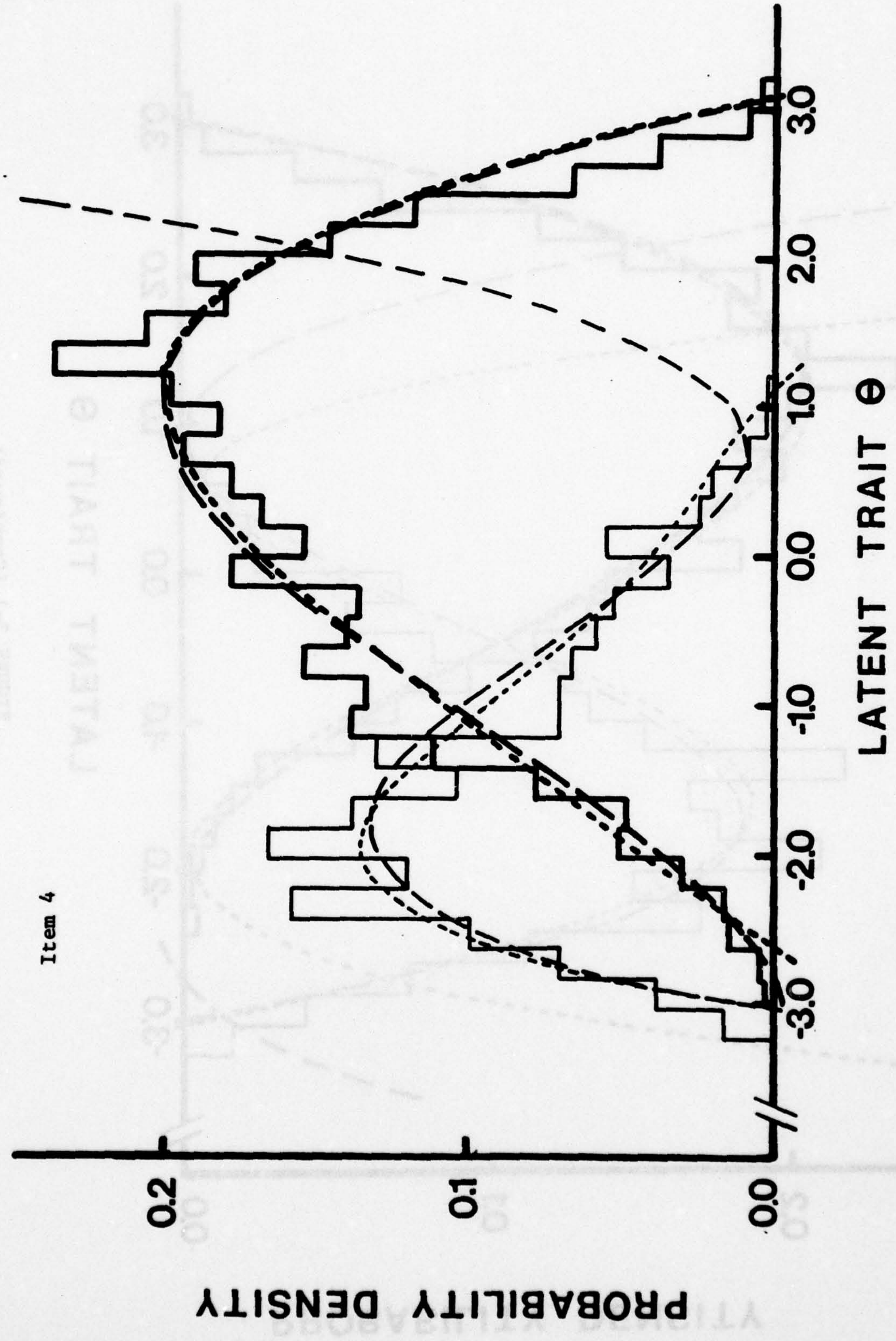

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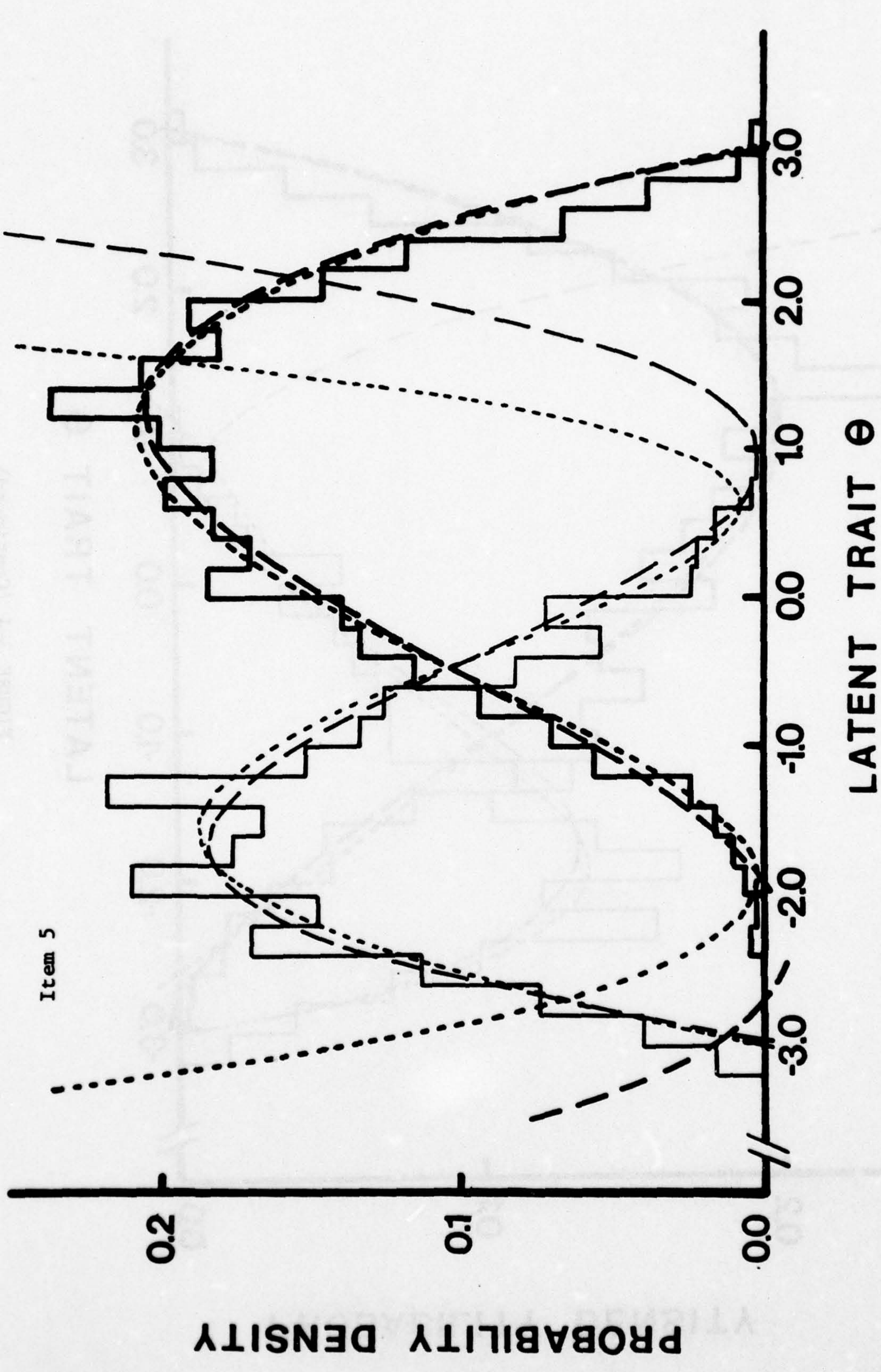

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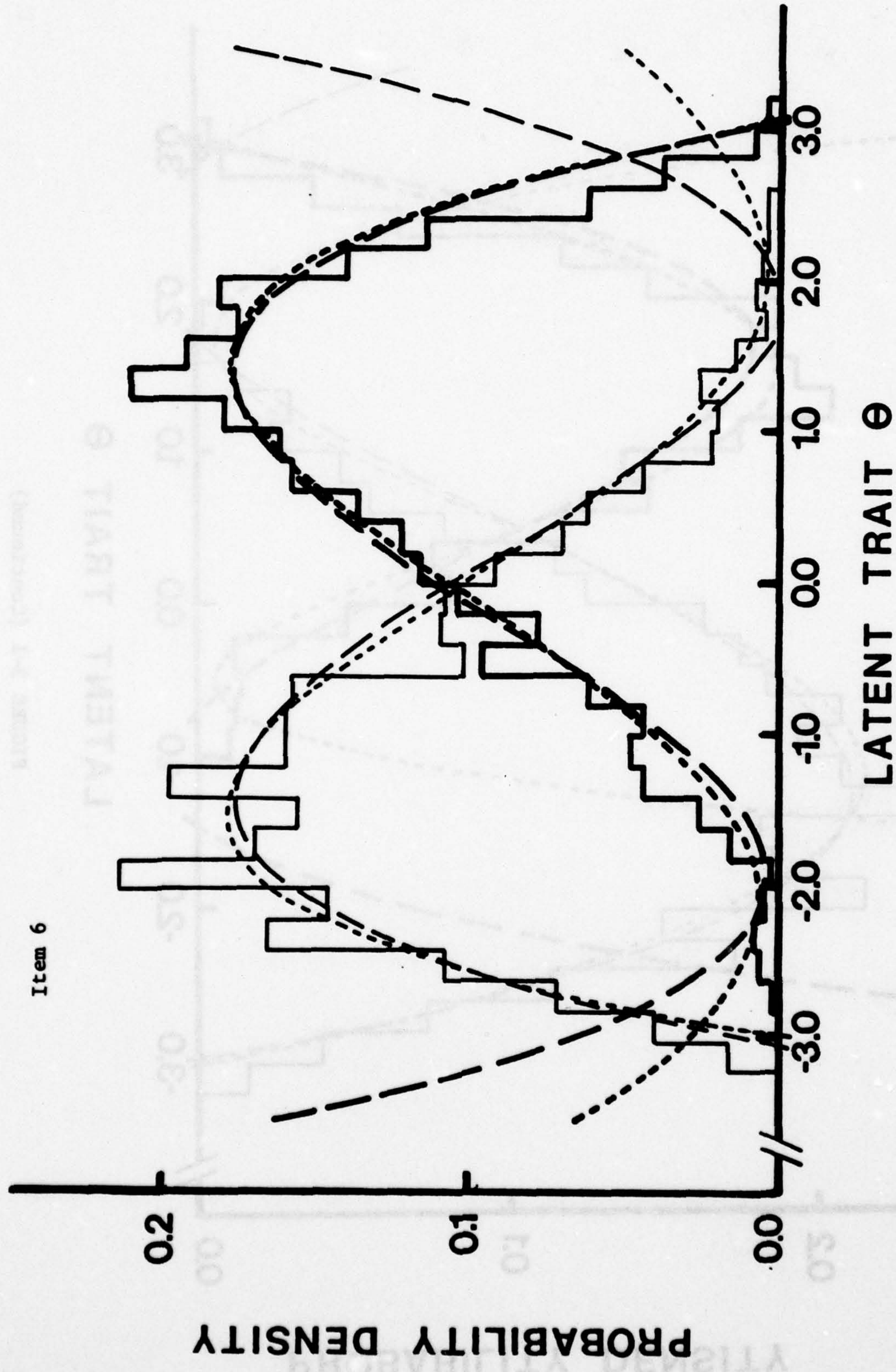

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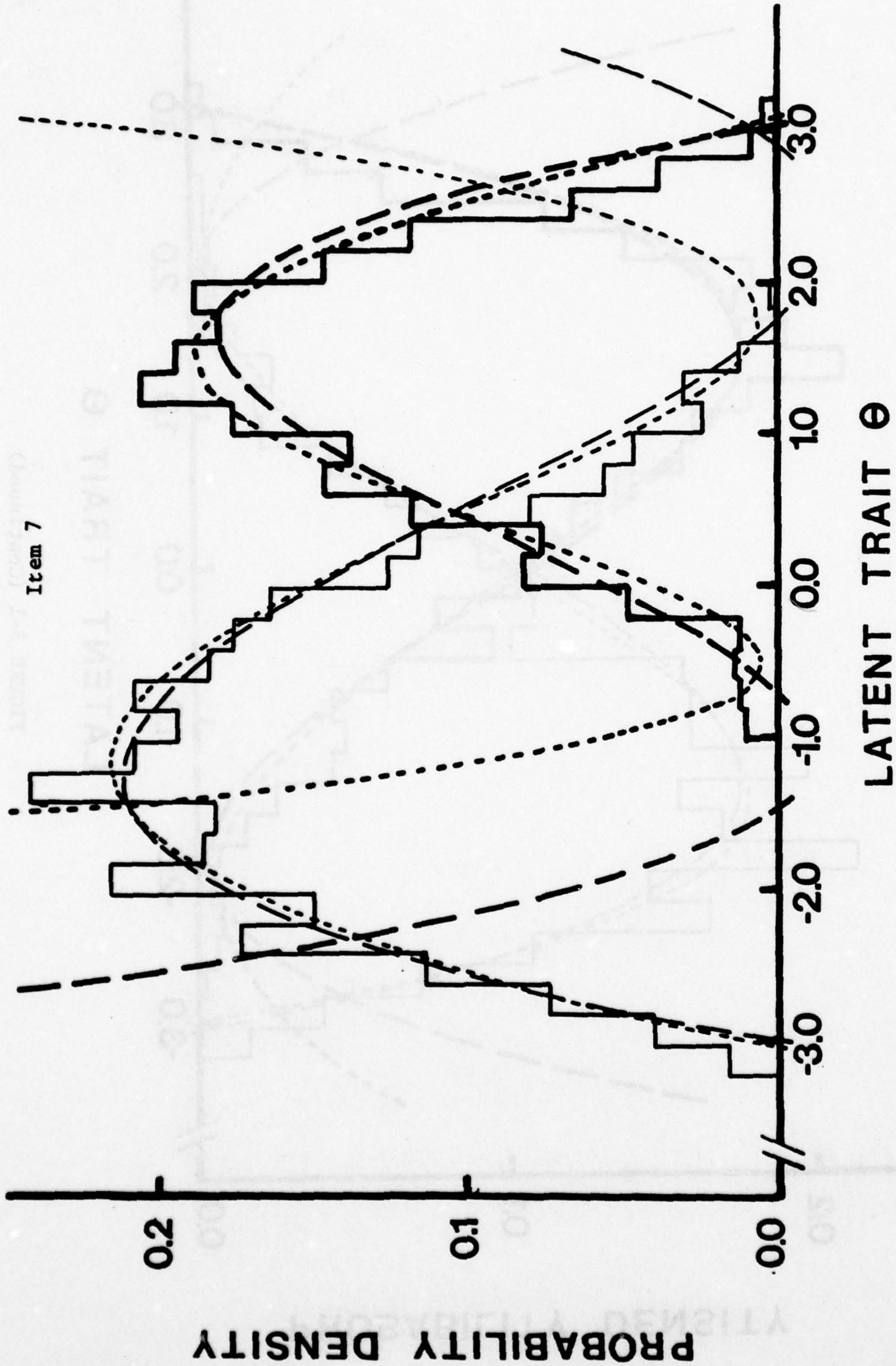

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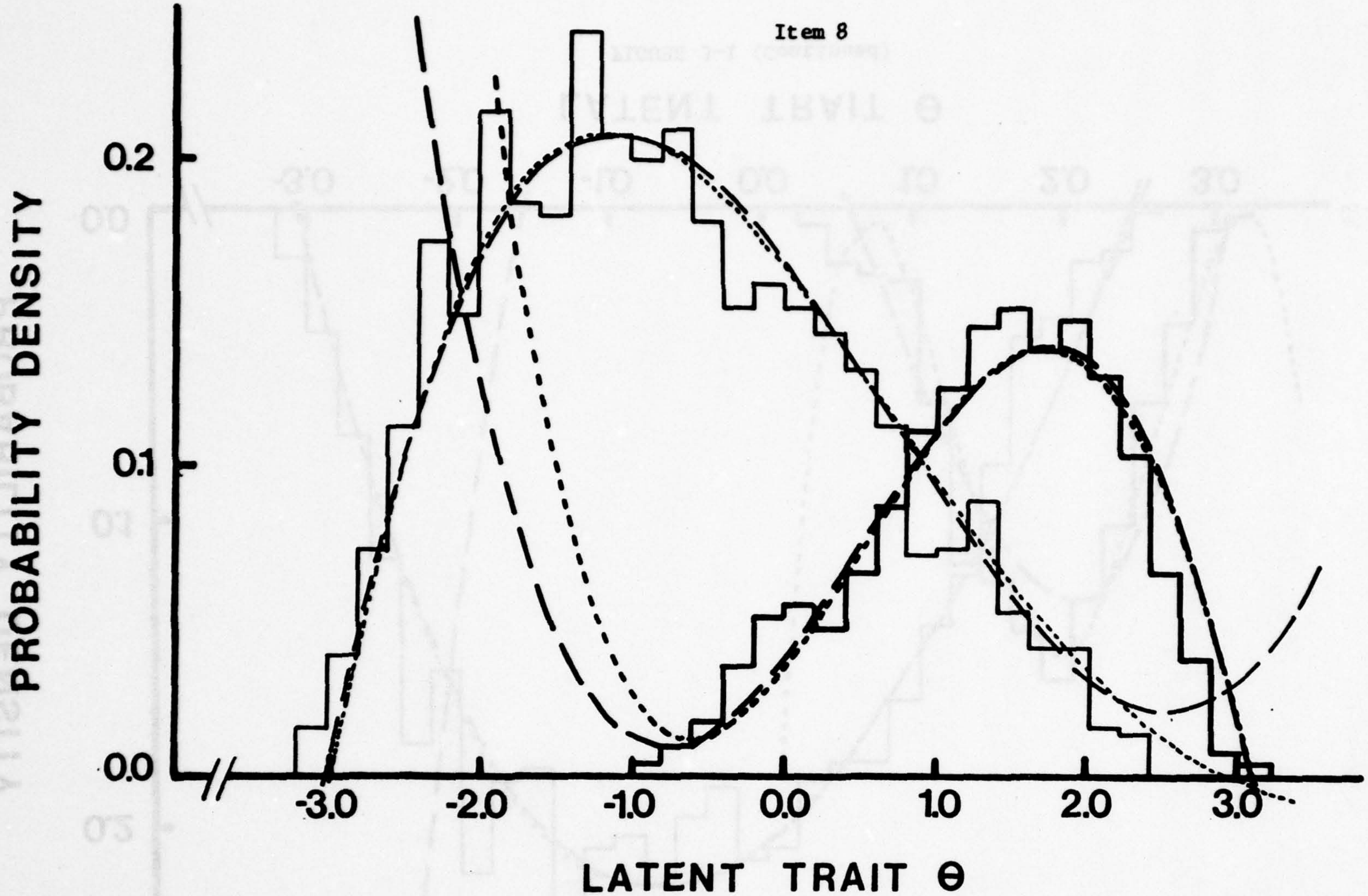

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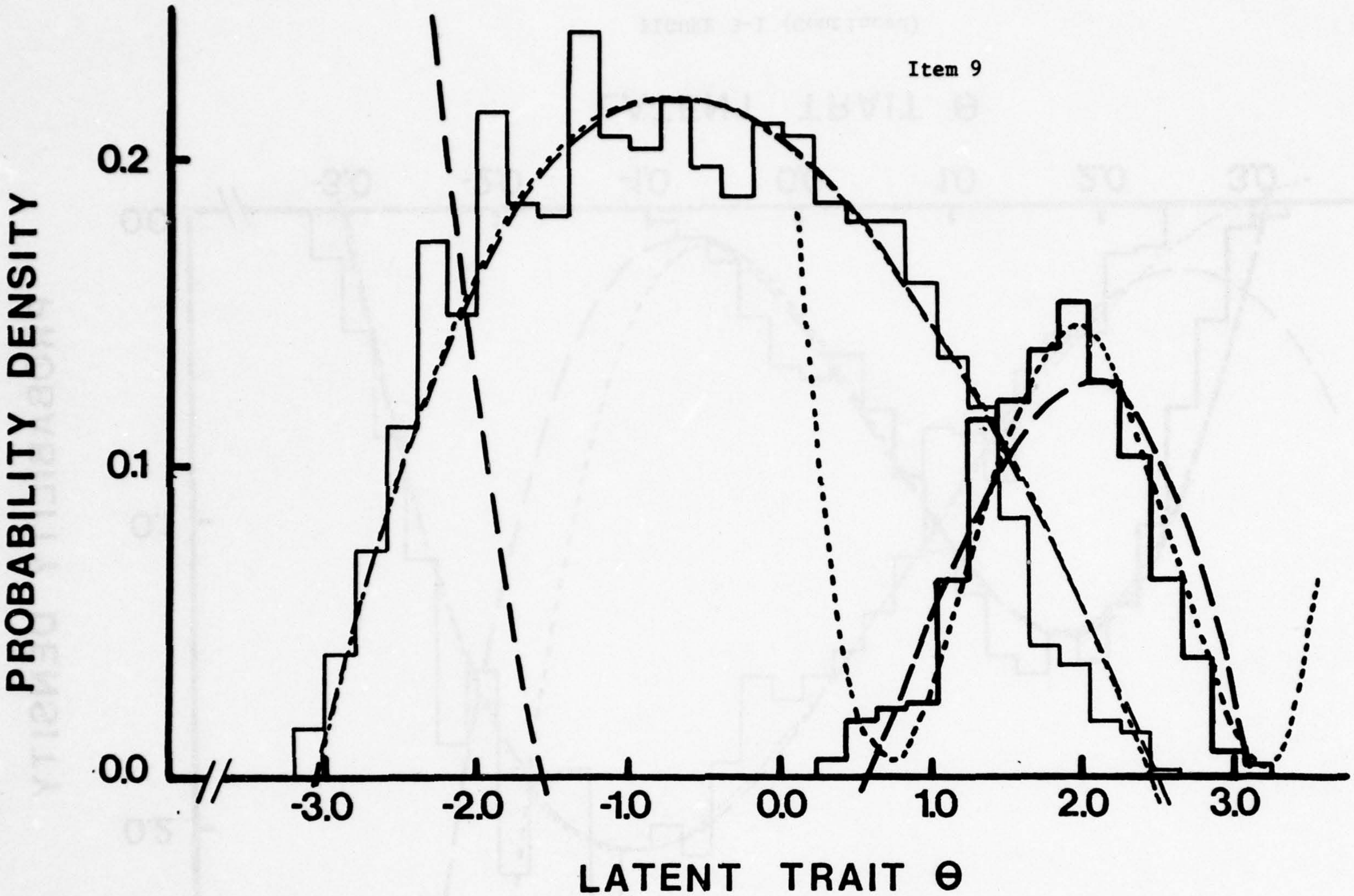

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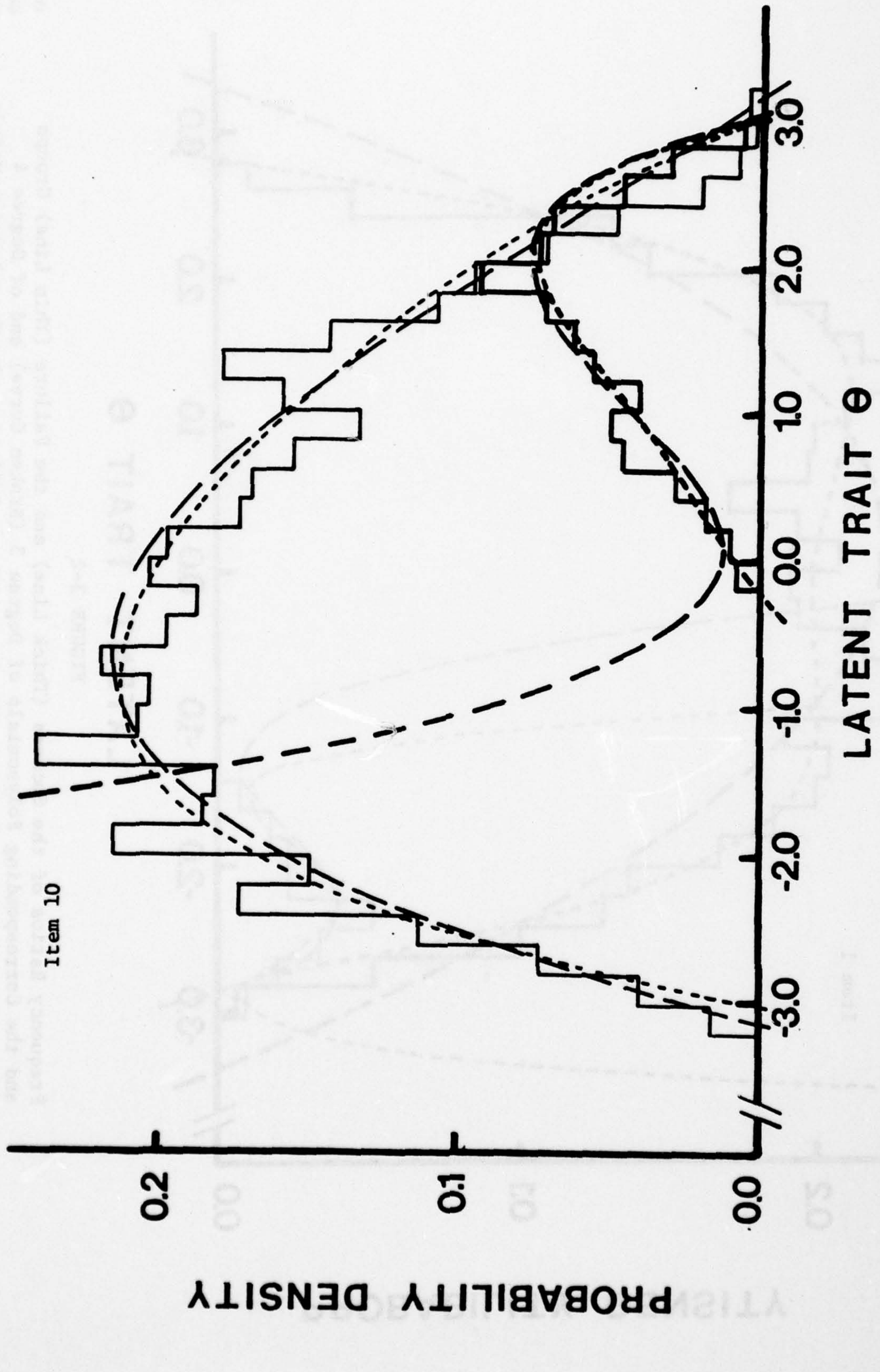

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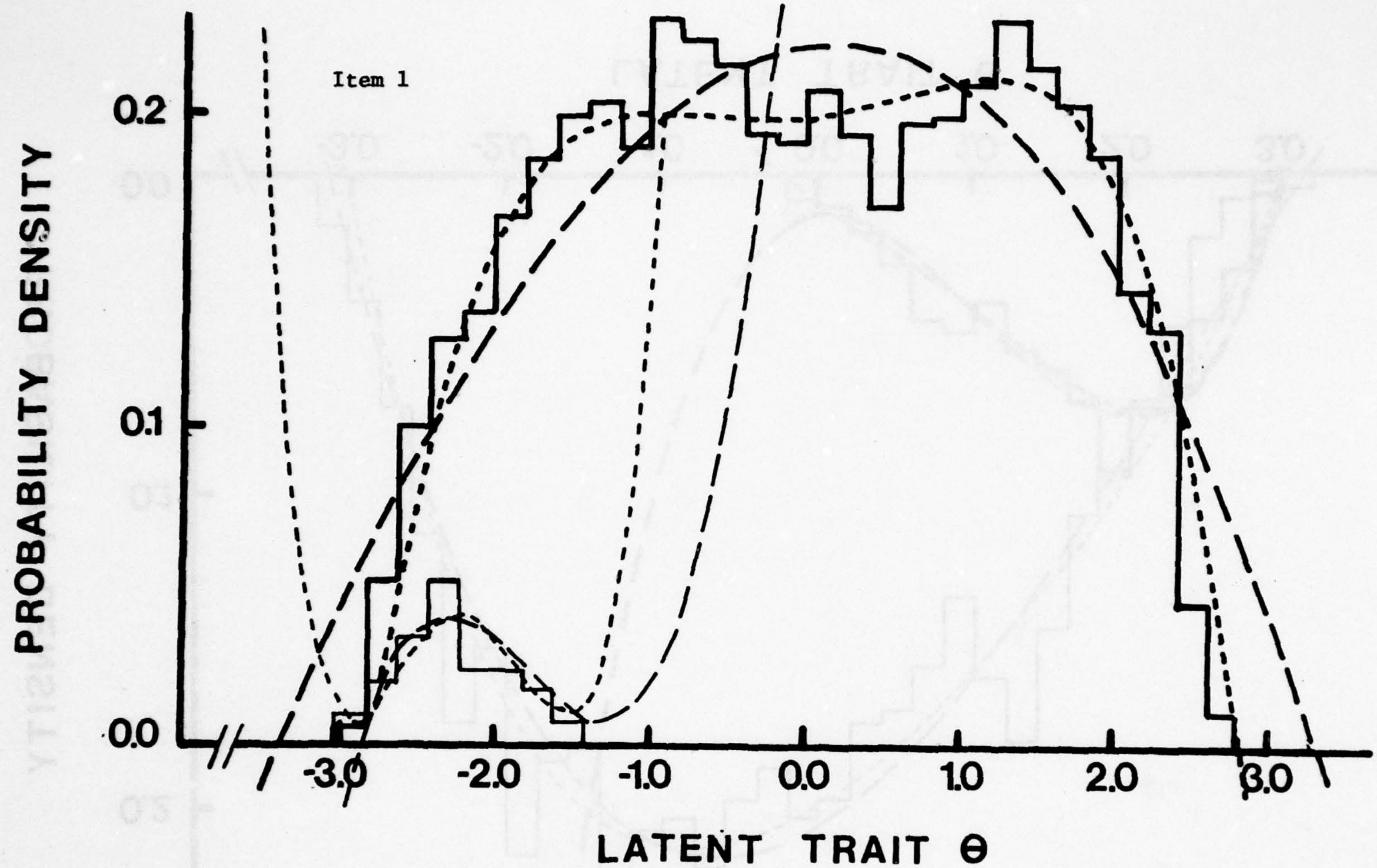
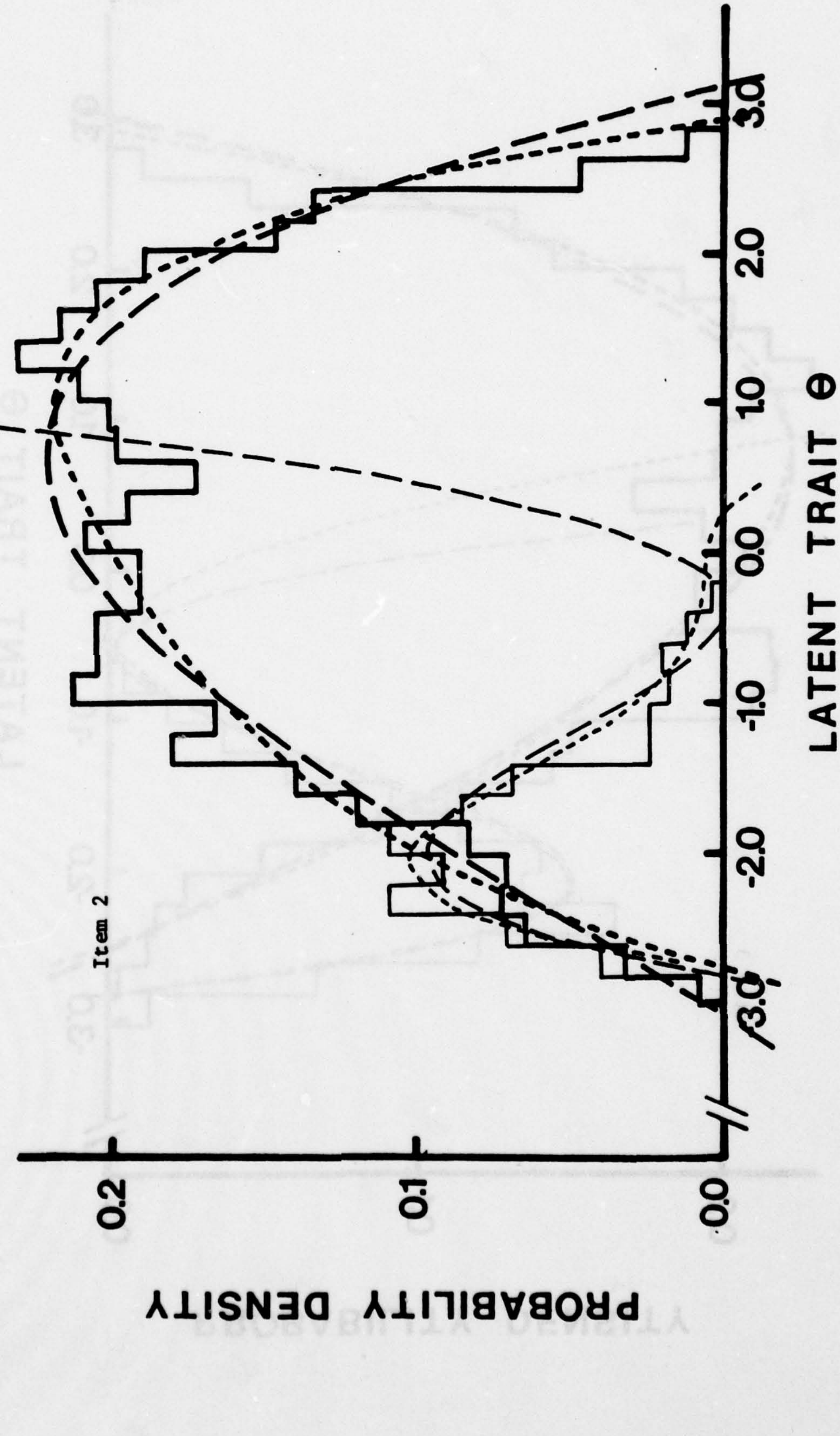


FIGURE 3-2

Frequency Ratios of the Success (Thick Line) and the Failure (Thin Line) Groups and the Corresponding Polynomials of Degree 3 (Broken Curve) and of Degree 4 (Dashed Curve) for Each of the Ten Binary Items Following the Normal Ogive Model: Degree 4-3 and 4-4 Cases



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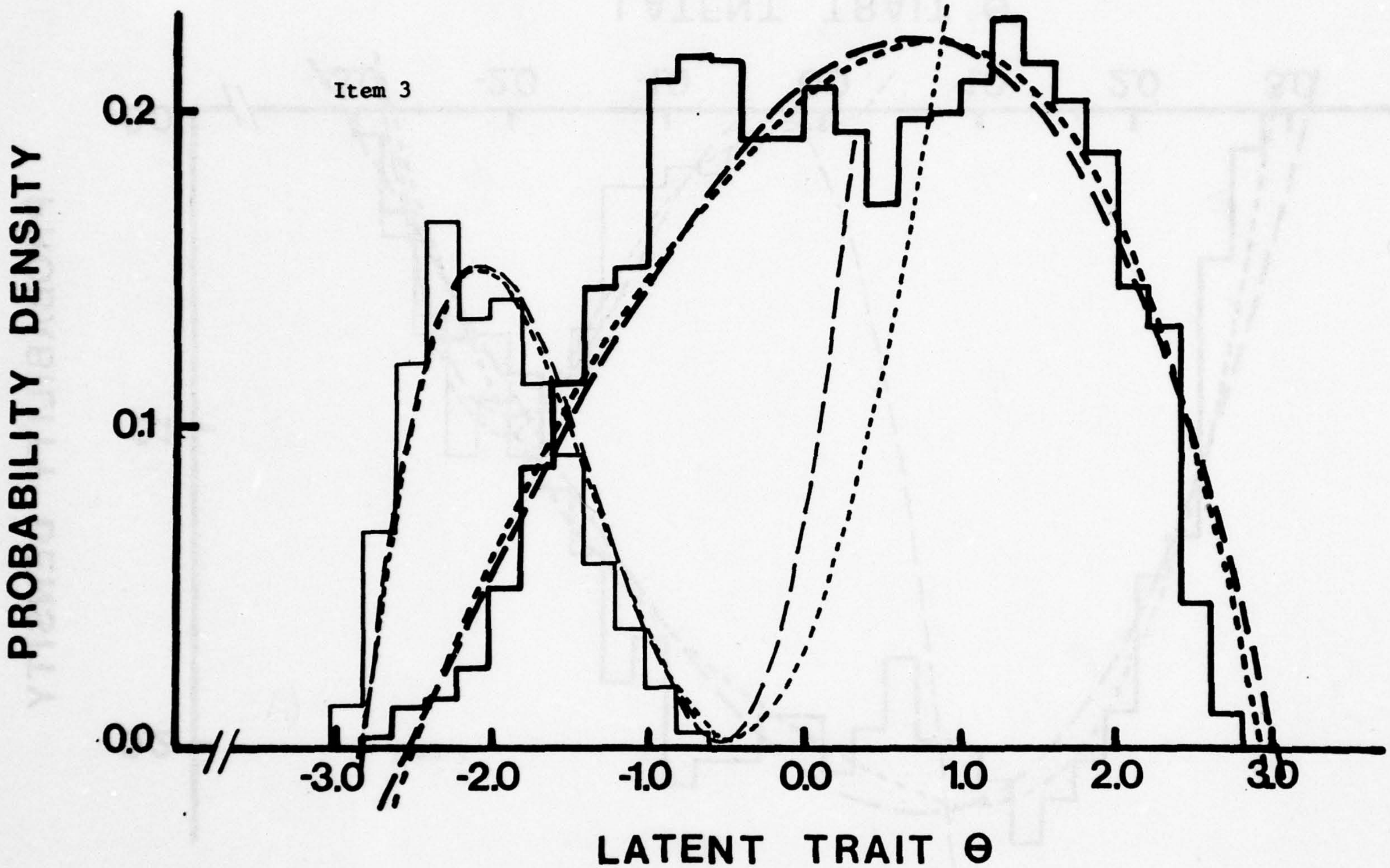

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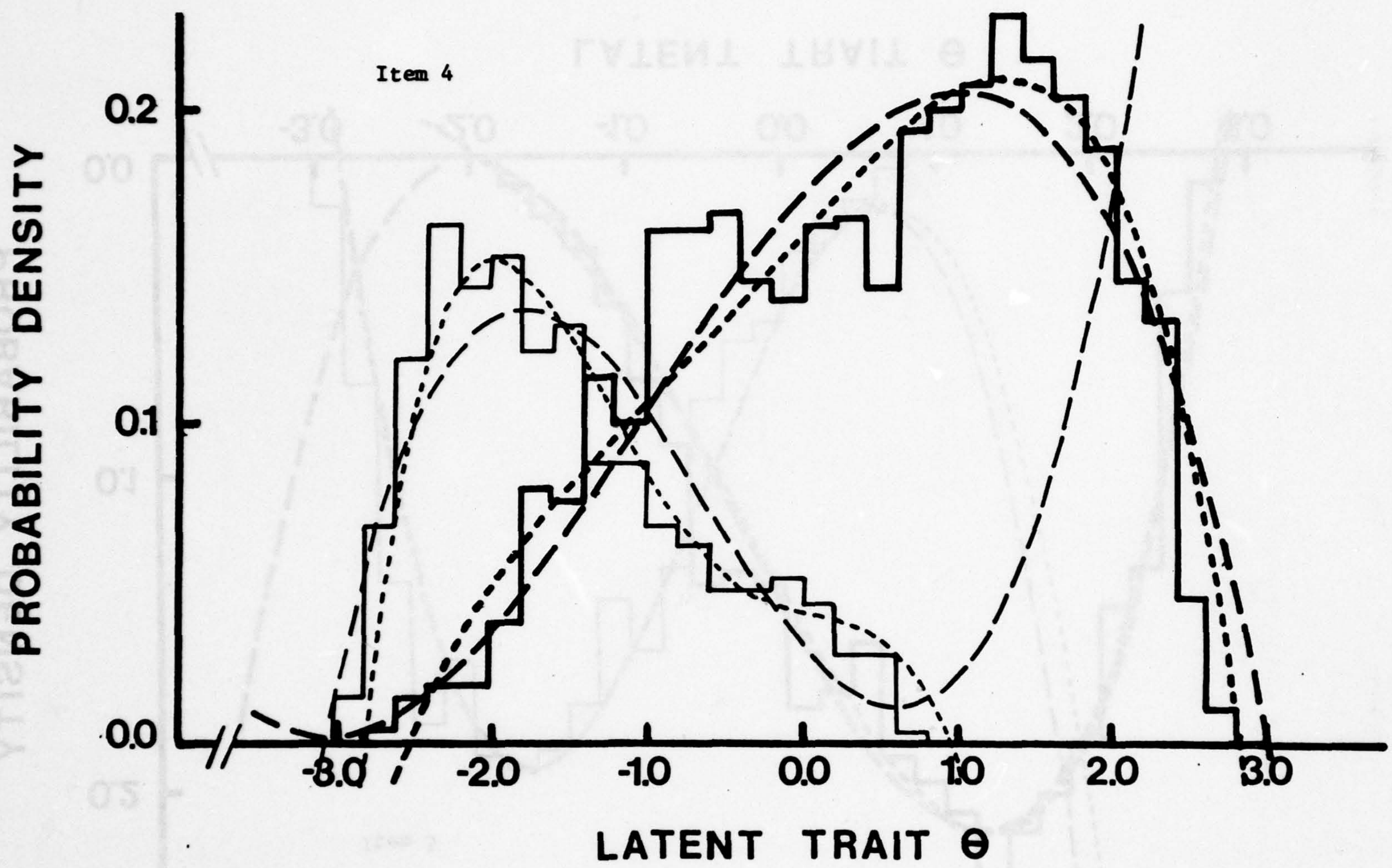

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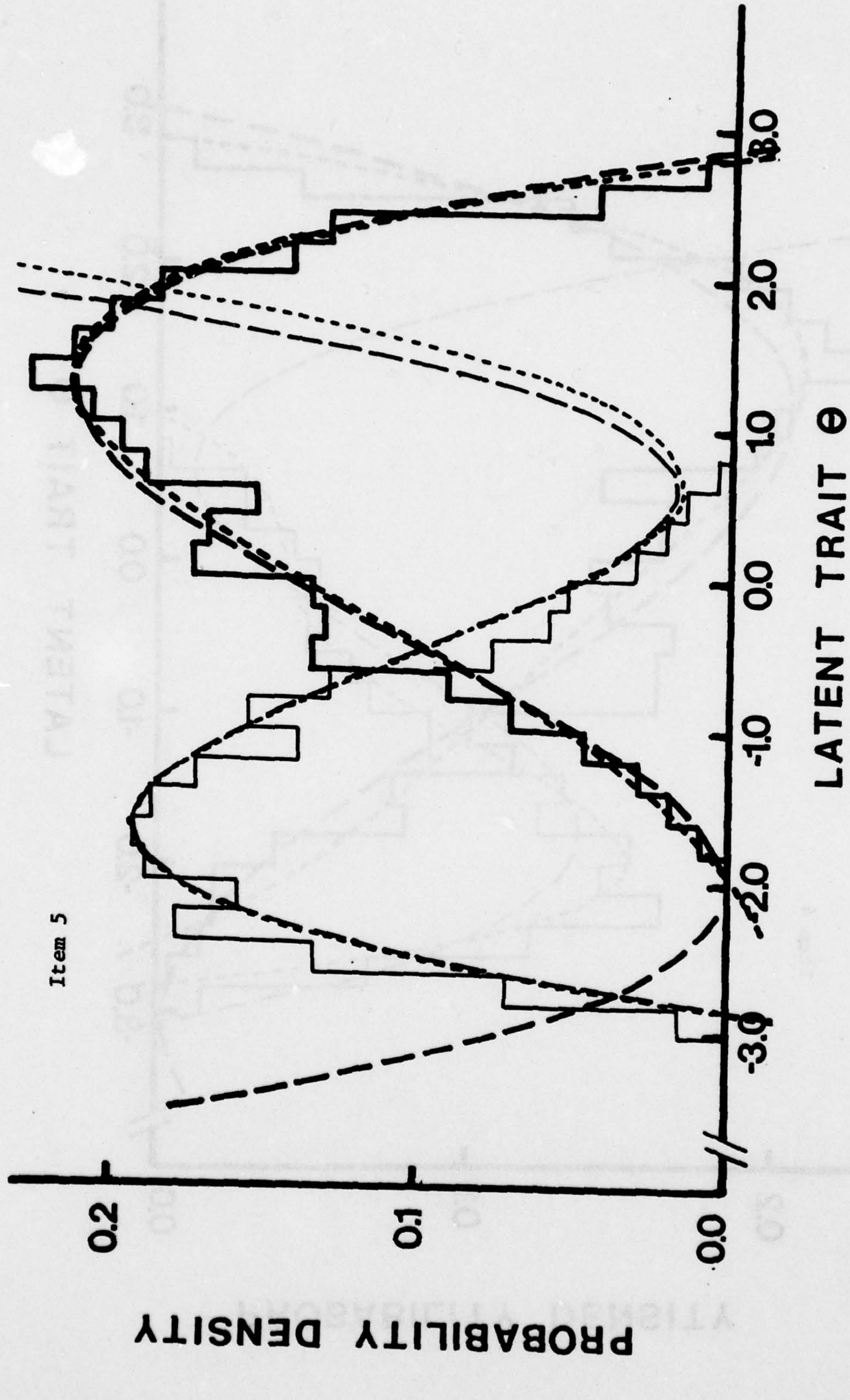

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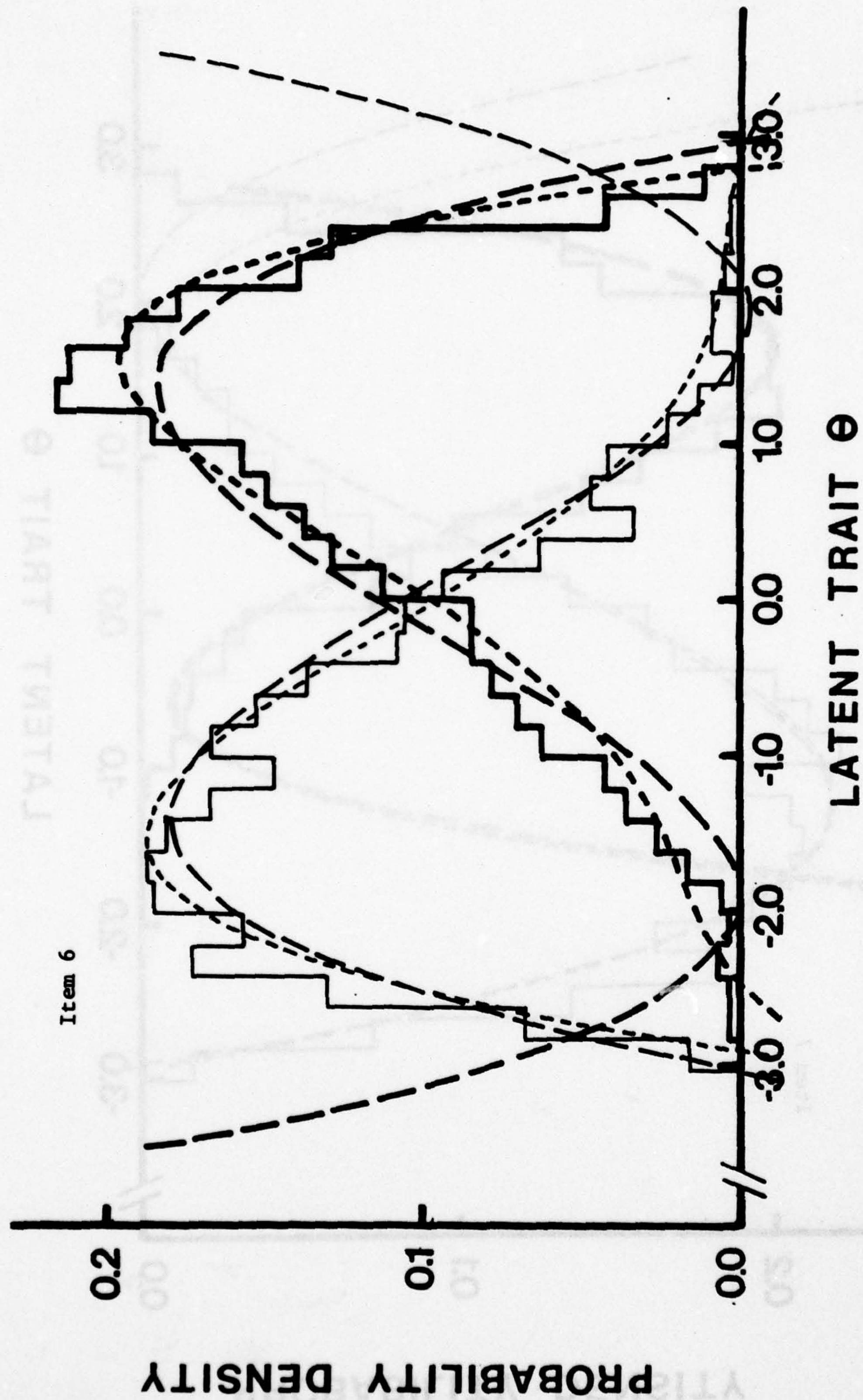

FIGURE 3-2 (Continued)

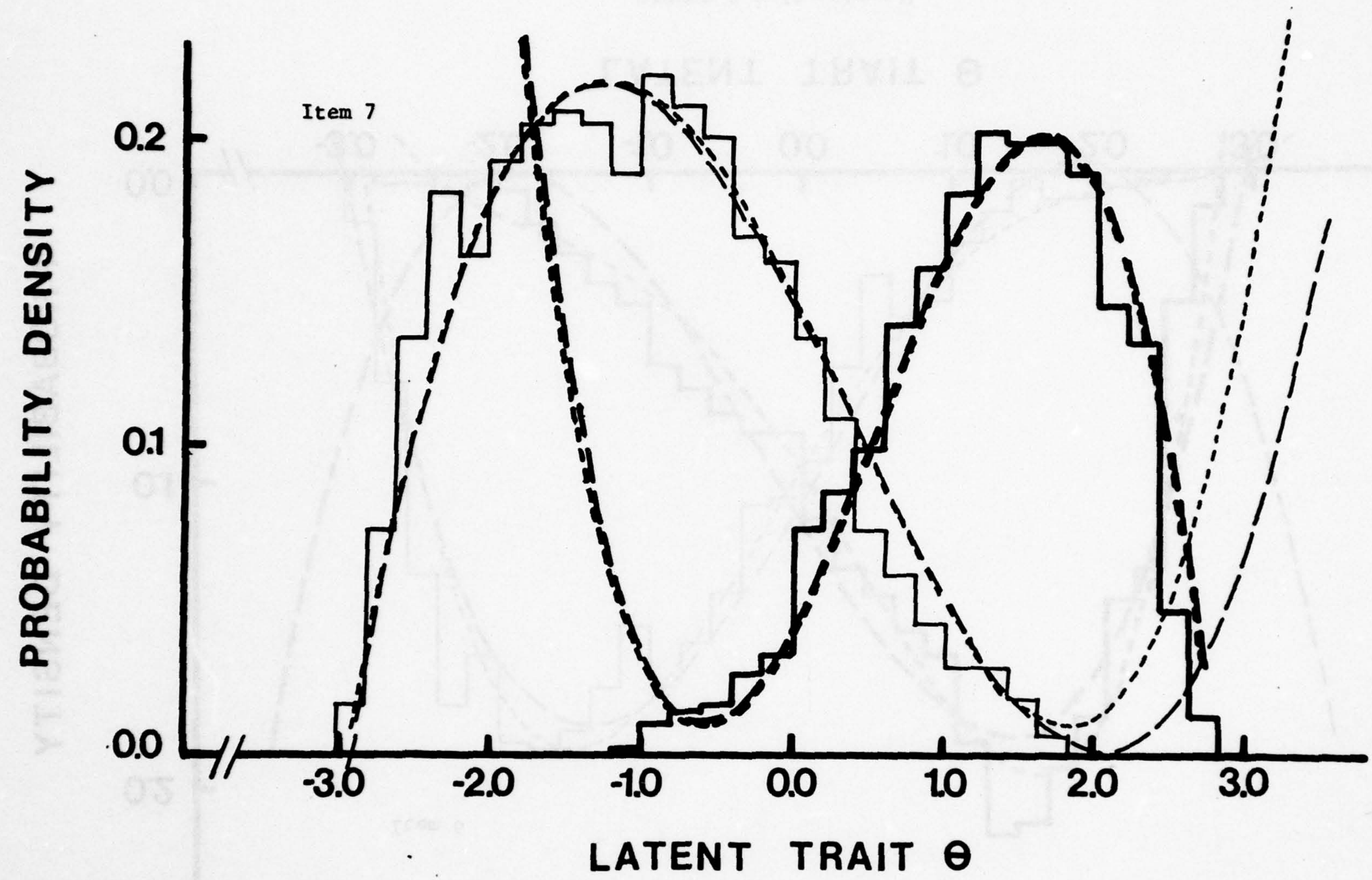

FIGURE 3-2 (Continued)

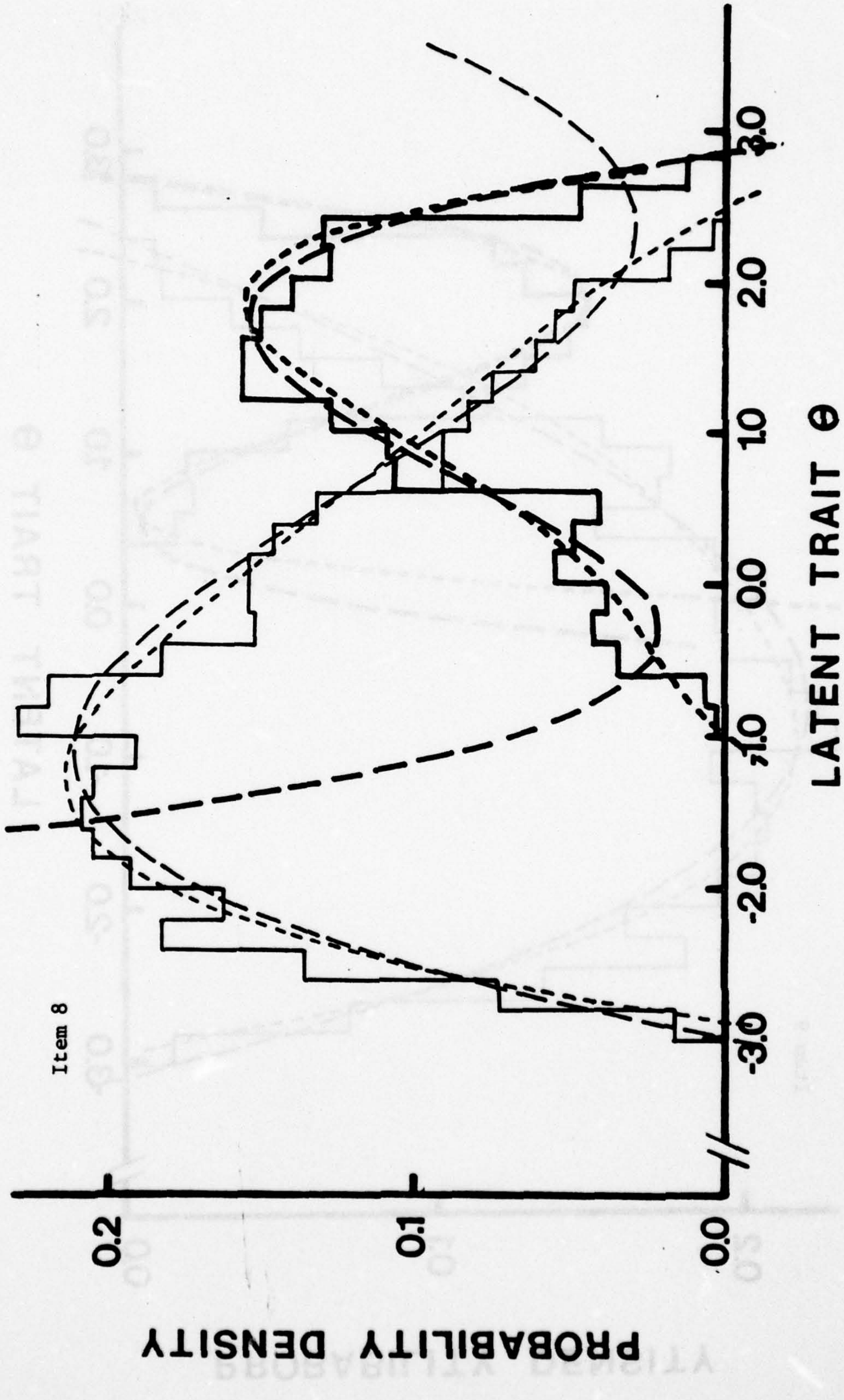

FIGURE 3-2 (Continued)

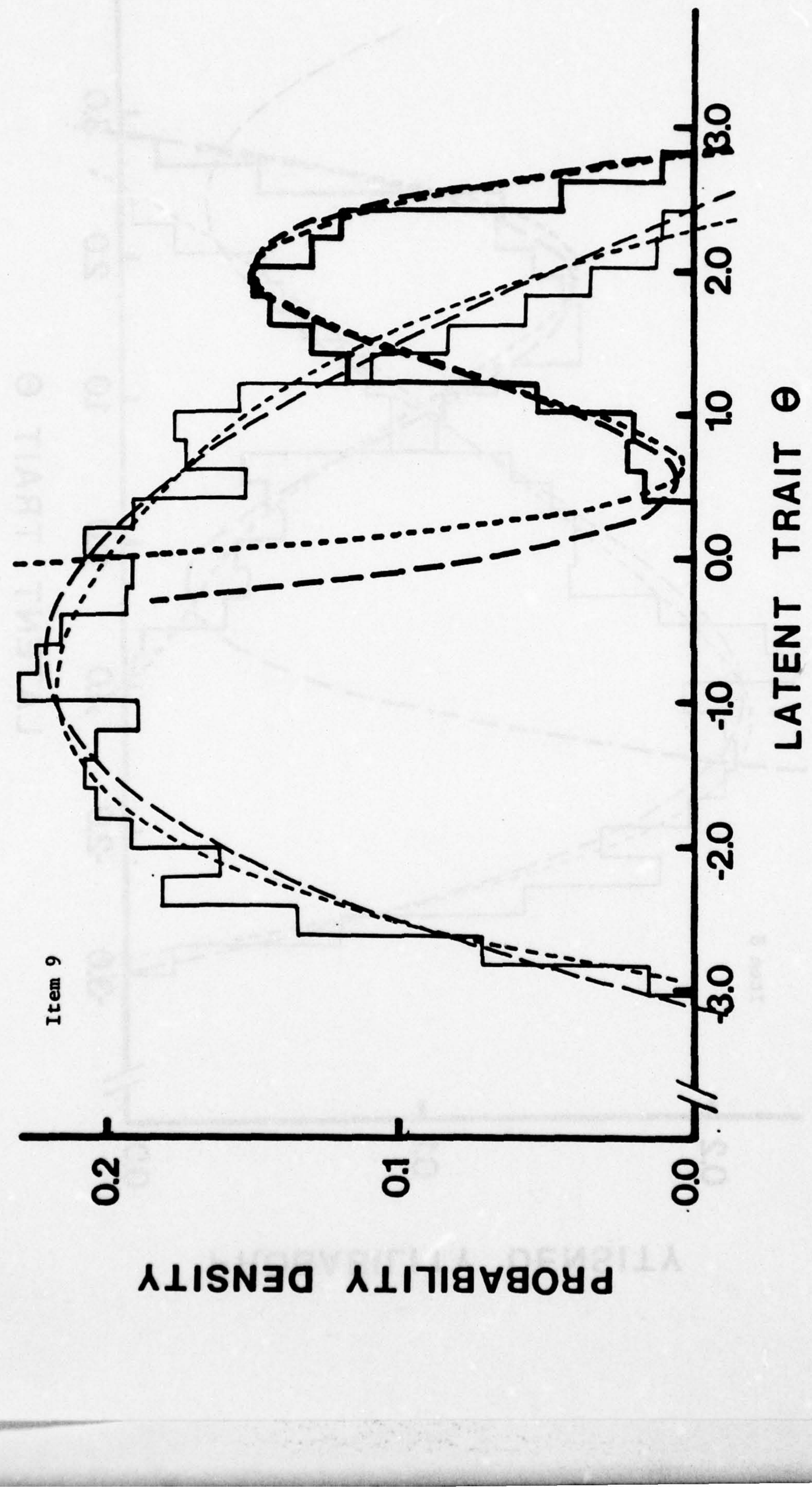

FIGURE 3-2 (Continued)

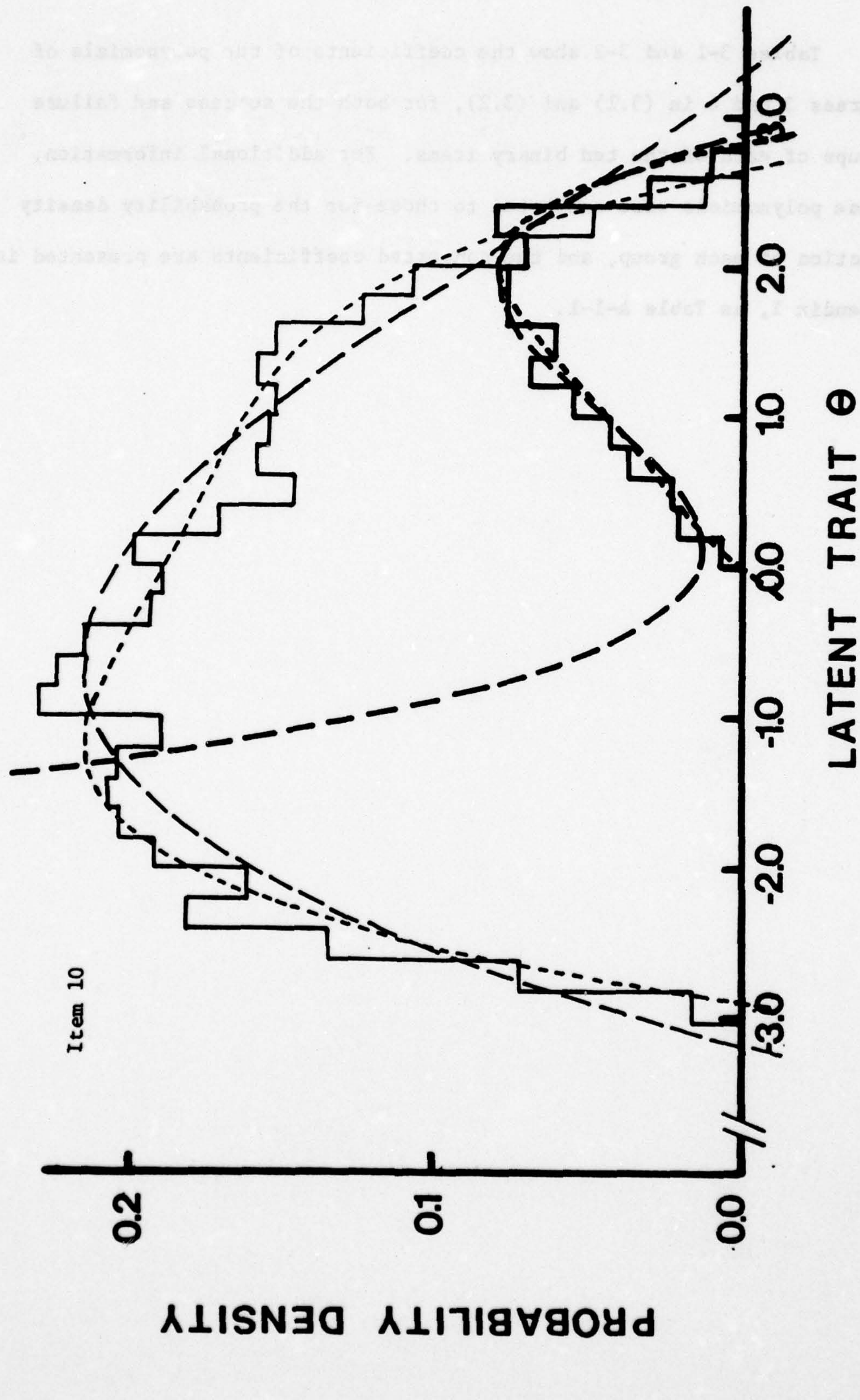

FIGURE 3-2 (Continued)

Tables 3-1 and 3-2 show the coefficients of the polynomials of degrees 3 and 4 in (3.1) and (3.2), for both the success and failure groups of each of the ten binary items. For additional information, these polynomials were converted to those for the probability density function of each group, and the converted coefficients are presented in Appendix I, as Table A-1-1.

TABLE 3-1

Estimated Coefficients of the Polynomial Graduating the set of $\tilde{\theta}$ of Each of the Failure and Success Groups of Examinees: Degree 3-3 Case

ITEM	FAILURE $\hat{\alpha}$	FAILURE $\hat{\beta}$	FAILURE $\hat{\gamma}$	FAILURE $\hat{\delta}$	SUCCESS $\hat{\alpha}$	SUCCESS $\hat{\beta}$	SUCCESS $\hat{\gamma}$	SUCCESS $\hat{\delta}$
1	-0.02516	0.01792	0.04995	0.01339	0.22694	0.00608	-0.02276	-0.00033
2	0.01541	0.07820	0.13690	0.03916	0.20885	0.02995	-0.02163	-0.00313
3	-0.04854	-0.02585	0.10734	0.03717	0.20799	0.03730	-0.02554	-0.00270
4	0.03430	-0.06104	0.02556	0.01615	0.17199	0.05353	-0.01840	-0.00556
5	0.05580	-0.09794	0.02245	0.02093	0.14426	0.08188	-0.01475	-0.00933
6	0.10665	-0.07975	-0.00699	0.01017	0.11166	0.07683	-0.00760	-0.00966
7	0.14823	-0.08709	-0.01647	0.00997	0.05425	0.09900	0.01695	-0.01850
8	0.16678	-0.06527	-0.01655	0.00777	0.03864	0.06565	0.02357	-0.01612
9	0.20732	-0.04066	-0.02579	0.00305	-0.06094	0.09441	0.04682	-0.02339
10	0.20709	-0.03044	-0.02141	0.00310	0.01239	-0.00691	0.05619	-0.01827

TABLE 3-1 (Continued)

Degree 3-4 Case

	FAILURE					SUCCESS				
ITEM	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{\nu}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{\nu}$
1	0.79083	1.75716	1.38805	0.45496	0.05291	0.20960	0.00339	-0.00510	0.00032	-0.00210
2	0.01779	0.09417	0.16134	0.05213	0.00222	0.20521	0.02943	-0.01795	-0.00301	-0.00044
3	0.34154	1.13515	1.25939	0.49924	0.06445	0.20780	0.03718	-0.02531	-0.00266	-0.00003
4	0.03961	-0.03966	0.01637	-0.00517	-0.00558	0.16717	0.05092	-0.01288	-0.00486	-0.00076
5	0.04825	-0.11287	0.04324	0.04692	0.00608	0.14896	0.09076	-0.02203	-0.01240	0.00153
6	0.10068	-0.07578	0.00069	0.00899	-0.00119	0.10776	0.07299	-0.00258	-0.00852	-0.00080
7	0.15214	-0.09857	-0.02406	0.01477	0.00215	0.03927	0.11822	0.06242	-0.06439	0.00980
8	0.16376	-0.06280	-0.01250	0.00700	-0.00066	0.03552	0.06895	0.03376	-0.02585	0.00205
9	0.20449	-0.03858	-0.02207	0.00242	-0.00059	0.26794	-0.96101	1.12990	-0.46148	0.06068
10	0.19836	-0.03170	-0.01255	0.00340	-0.00106	0.00790	0.03161	-0.00688	0.01617	-0.00596

TABLE 3-2

Estimated Coefficients of the Polynomial Graduating the Set of $\hat{\theta}$ of Each of the Failure and Success Groups of Examinees: Degree 4-3 Case

ITEM	FAILURE				SUCCESS			
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\delta}$
1	0.29923	0.56476	0.34633	0.06512	0.22550	0.00769	-0.02035	-0.00083
2	0.01773	0.10104	0.16086	0.04523	0.21303	0.03108	-0.02253	-0.00323
3	0.07028	0.28057	0.33197	0.08577	0.21084	0.04103	-0.02591	-0.00354
4	0.03227	-0.05974	0.03091	0.01784	0.17543	0.05682	-0.01917	-0.00625
5	0.04923	-0.09109	0.04881	0.03053	0.14679	0.09391	-0.01457	-0.01265
6	0.10818	-0.08277	-0.00722	0.01071	0.11487	0.08362	-0.00877	-0.01103
7	0.14770	-0.09391	-0.01432	0.01235	0.04075	0.09966	0.05665	-0.03526
8	0.16828	-0.06798	-0.01626	0.00851	0.03065	0.05256	0.06167	-0.02958
9	0.20980	-0.04010	-0.02614	0.00278	0.07107	-0.27577	0.33550	-0.08873
10	0.20768	-0.03082	-0.02042	0.00324	0.01396	-0.01689	0.07174	-0.02338

TABLE 3-2 (Continued)

Degree 4-4 Case

ITEM	FAILURE $\hat{\alpha}$	FAILURE $\hat{\beta}$	FAILURE $\hat{\gamma}$	FAILURE $\hat{\delta}$	FAILURE $\hat{\nu}$	SUCCESS $\hat{\alpha}$	SUCCESS $\hat{\beta}$	SUCCESS $\hat{\gamma}$	SUCCESS $\hat{\delta}$	SUCCESS $\hat{\nu}$
1	2.66625	5.41868	3.99943	1.26203	0.14422	0.20079	0.00594	0.01177	-0.00030	-0.00487
2	0.00498	-0.00803	-0.02865	-0.06483	-0.02034	0.20071	0.03165	-0.00703	-0.00340	-0.00227
3	0.02919	0.12015	0.13729	-0.00592	-0.01466	0.20528	0.04058	-0.01867	-0.00341	-0.00110
4	0.04194	-0.01089	0.01361	-0.03495	-0.01529	0.16198	0.05587	-0.00168	-0.00597	-0.00265
5	0.06029	-0.08978	0.04442	0.02575	-0.00114	0.14246	0.08943	-0.00739	-0.01094	-0.00147
6	0.09786	-0.07928	0.00687	0.00960	-0.00226	0.09872	0.07963	0.01327	-0.00977	-0.00352
7	0.14926	-0.09793	-0.01761	0.01422	0.00097	0.04118	0.09850	0.05537	-0.03326	-0.00053
8	0.15975	-0.06256	-0.00368	0.00666	-0.00222	0.03895	0.04364	0.02452	0.01000	-0.00946
9	0.20100	-0.03374	-0.01288	0.00055	-0.00241	0.18819	-0.69498	0.82027	-0.31098	0.03501
10	0.19031	-0.02727	0.00242	0.00215	-0.00352	0.00068	0.06327	-0.04940	0.04181	-0.01142

The estimated item characteristic functions of the ten binary items in Degree 3 Case are presented as Figure 3-3, and those in Degree 4 Case are as Figure 3-4. In each case, the approximated polynomials of degree 3 are drawn by broken curves, and those of degree 4 are drawn by dotted curves.

As is expected, these curves pass through the plots obtained by the Histogram Ratio Method of $\tilde{\theta}$ in both Degree 3 Case (triangles) and Degree 4 Case (squares). We notice that there are crooked parts at the ends of the curves for some items, and this suggests the effect of the meaningless parts of the approximated polynomials, which are shown in Figures 3-1 and 3-2.

For additional information, a similar procedure has been applied for the set of 500 true θ and also for the set of 500 maximum likelihood estimates $\hat{\theta}$. The resulting curves are presented in Appendix I as Figures A-1-1 and A-1-2. Note in these cases the results of using polynomials of degree 5 are added to those of degrees 3 and 4.

Tables 3-3 and 3-4 present the estimates of the two parameters of the normal ogive model, i.e., the discrimination parameter a_g and the difficulty parameter b_g , which were obtained by the simple least square method described in the preceding section. In order to avoid the effect of the meaningless parts of the approximated polynomials, a rather narrow range of θ , -2.0 through 2.0 , is taken for the estimation, with the subinterval step of 0.2 , and within this interval the values of the estimated item characteristic function between 0.05 and 0.95 inclusive were used. We can see that the results of the present method are as good as those obtained from the true θ in both parameters. Since the strict

restriction on the range of θ has resulted in reducing the number of points used for the estimation, and indeed for item 1 on the true θ using the polynomials of degree 4 there is only one point and the estimates are not obtainable, similar parameter estimates using the range of the estimated item characteristic function 0.01 through 0.99 inclusive, instead of 0.05 through 0.95, were obtained, and presented as Tables 3-5 and 3-6. In these tables, we can see that the results of the present method are even better, compared with those for the true θ, especially in the estimation of a_g. For an additional information, the corresponding results obtained by using the interval of θ between -3.0 and 3.0 inclusive are presented as Tables 3-7 and 3-8. We notice that the estimates of the present method are much worse, especially for the discrimination parameter a_g, which indicate the effect of the crooked parts of the estimated item characteristic functions.

The comparison of the results of the present method with the corresponding results by the Histogram Ratio Method of the Two-Parameter Beta Method for $\tilde{\theta}$ indicates that, as a whole, there is some improvement in the estimation of the discrimination parameter a_g, but no visible improvement in the estimation of the difficulty parameter b_g (cf. Samejima, 1977d, Tables 7-1, 7-2, A-3-1 and A-3-2).

There are some mistakes in the plotting of the triangles, which represent the results of Degree 3 Case by the Histogram Ratio Method, in Figure 7-9 of the Research Report 77-1. Please refer to Figure 3-3 of this research report for corrections.

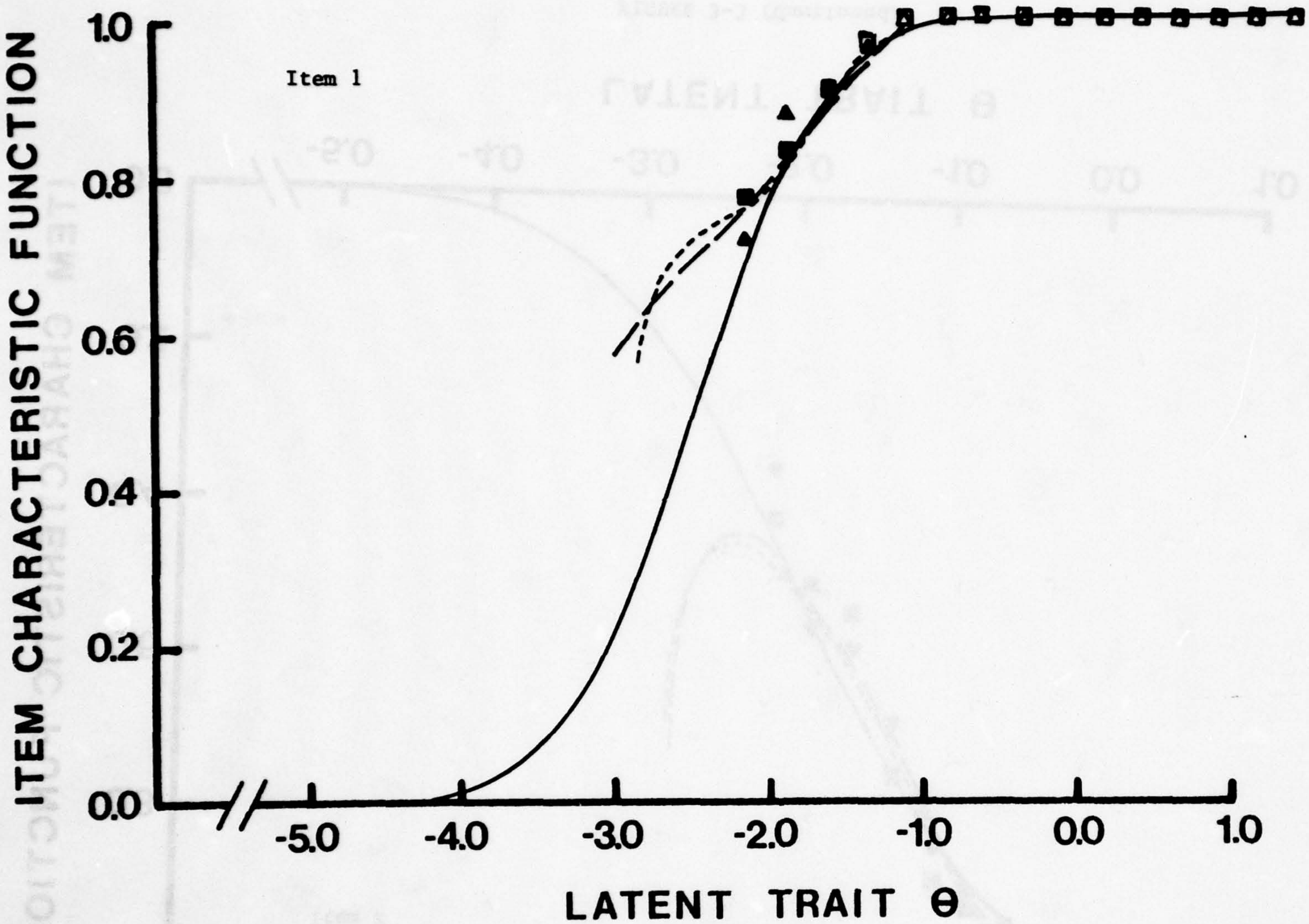

FIGURE 3-3

Estimated Item Characteristic Functions by the Curve Fitting Method for $\tilde{\theta}$, in Degree 3-3 Case (Broken Curve) and Degree 3-4 Case (Dashed Curve), with Those Obtained by the Histogram Ratio Method for $\tilde{\theta}$ in Degree 3 Case (Triangles) and Degree 4 Case (Squares) and the True Item Characteristic Function (Solid Curve)

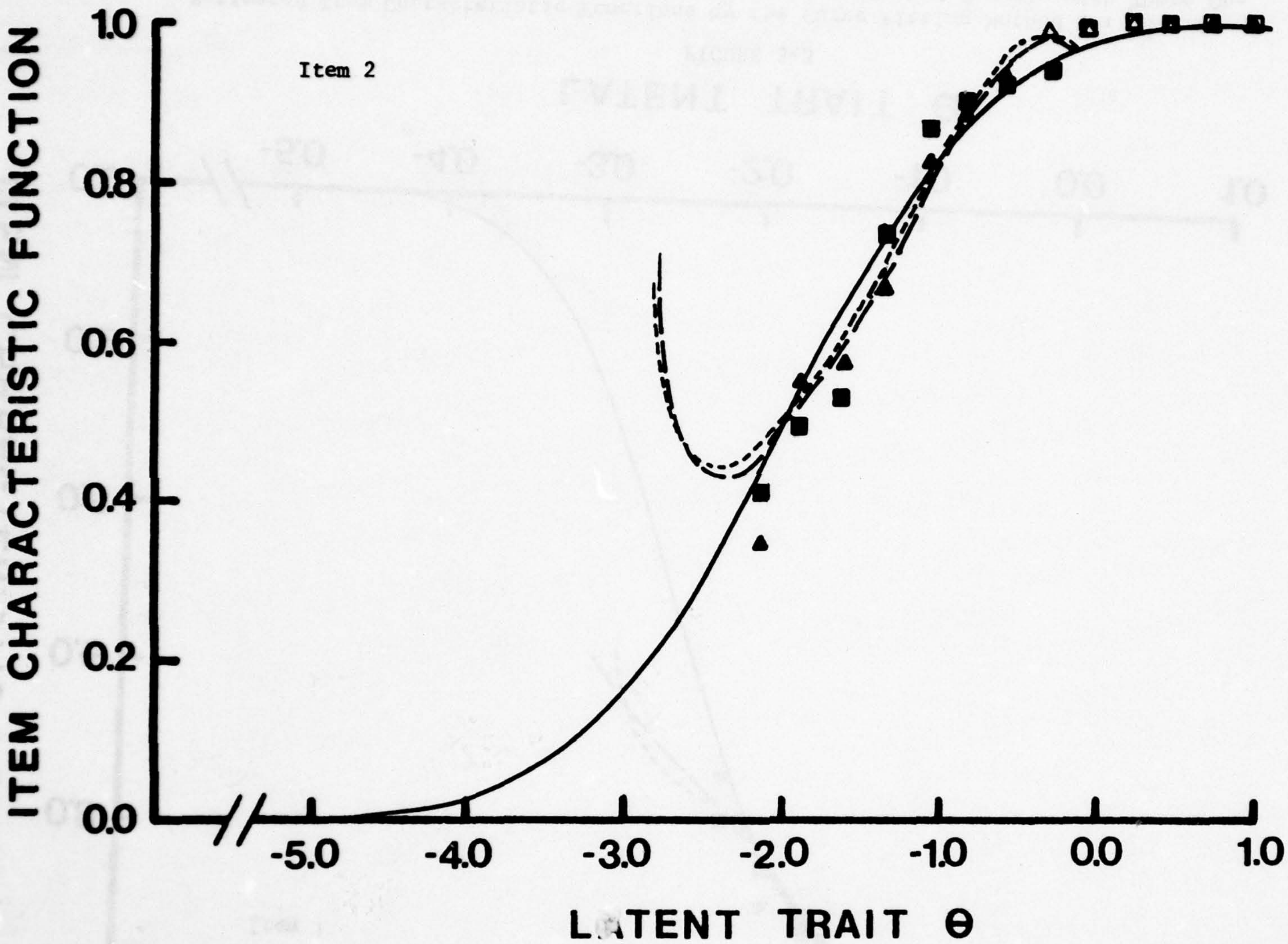

FIGURE 3-3 (Continued)

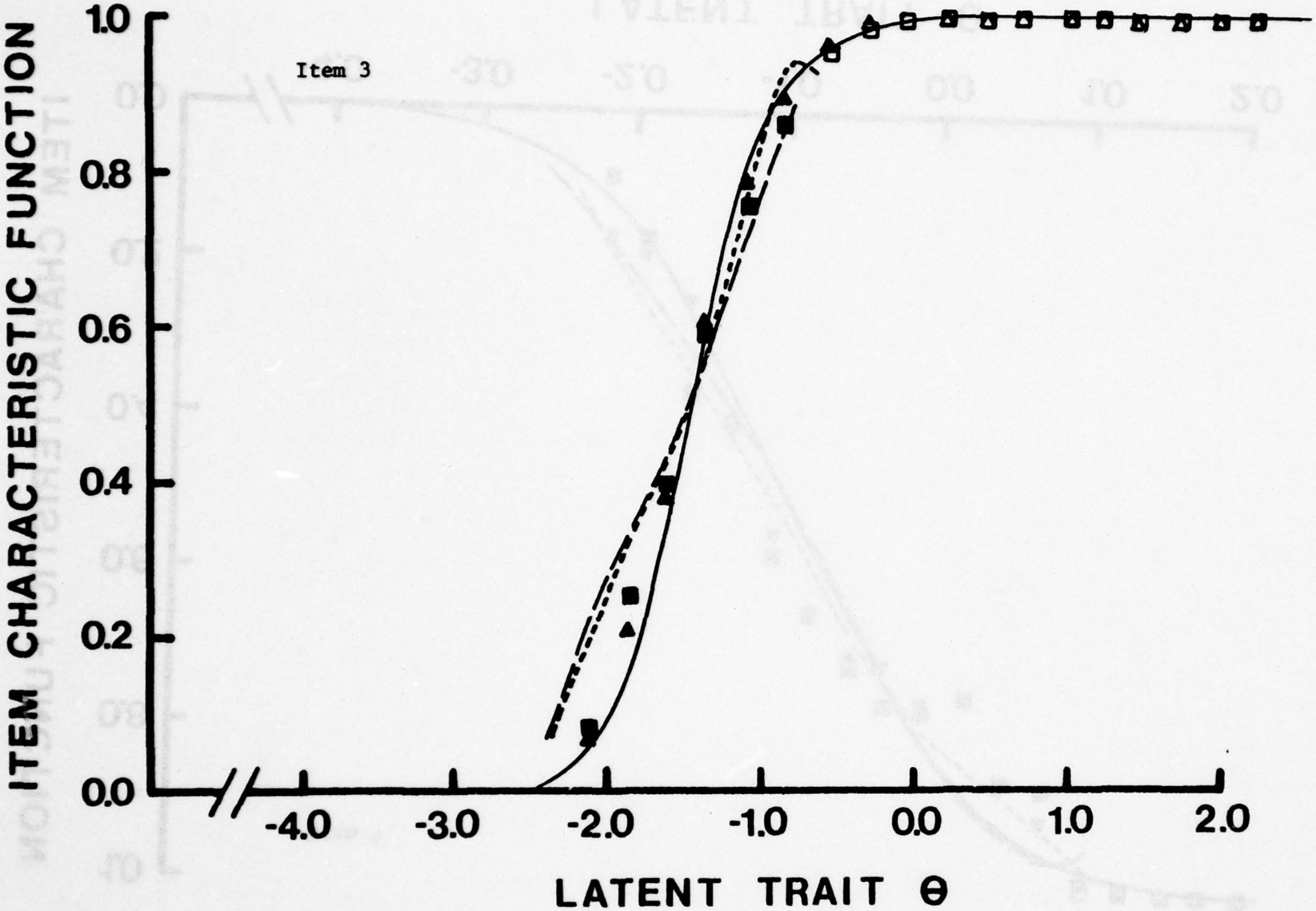

FIGURE 3-3 (Continued)

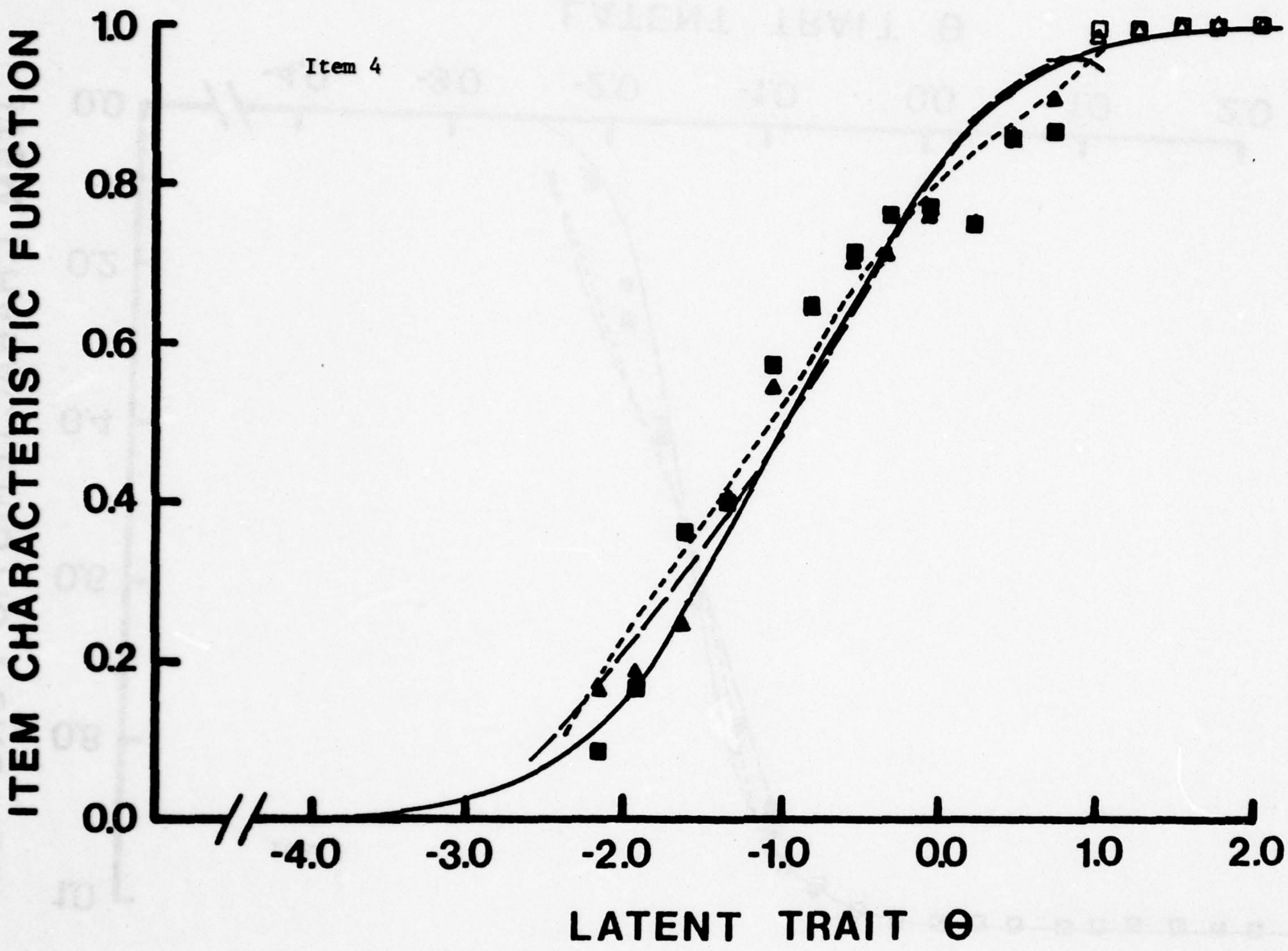

FIGURE 3-3 (Continued)

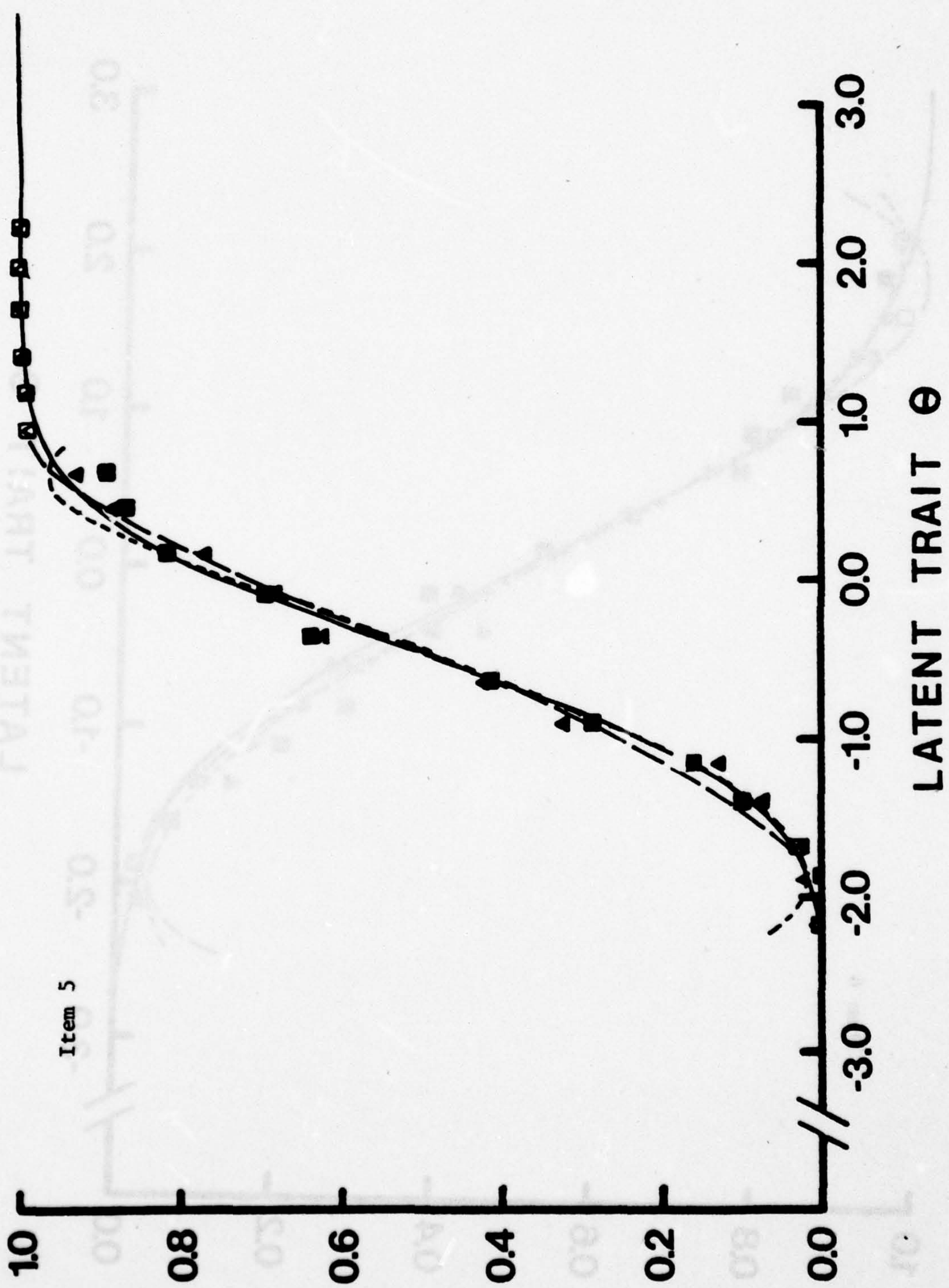

FIGURE 3-3 (Continued)

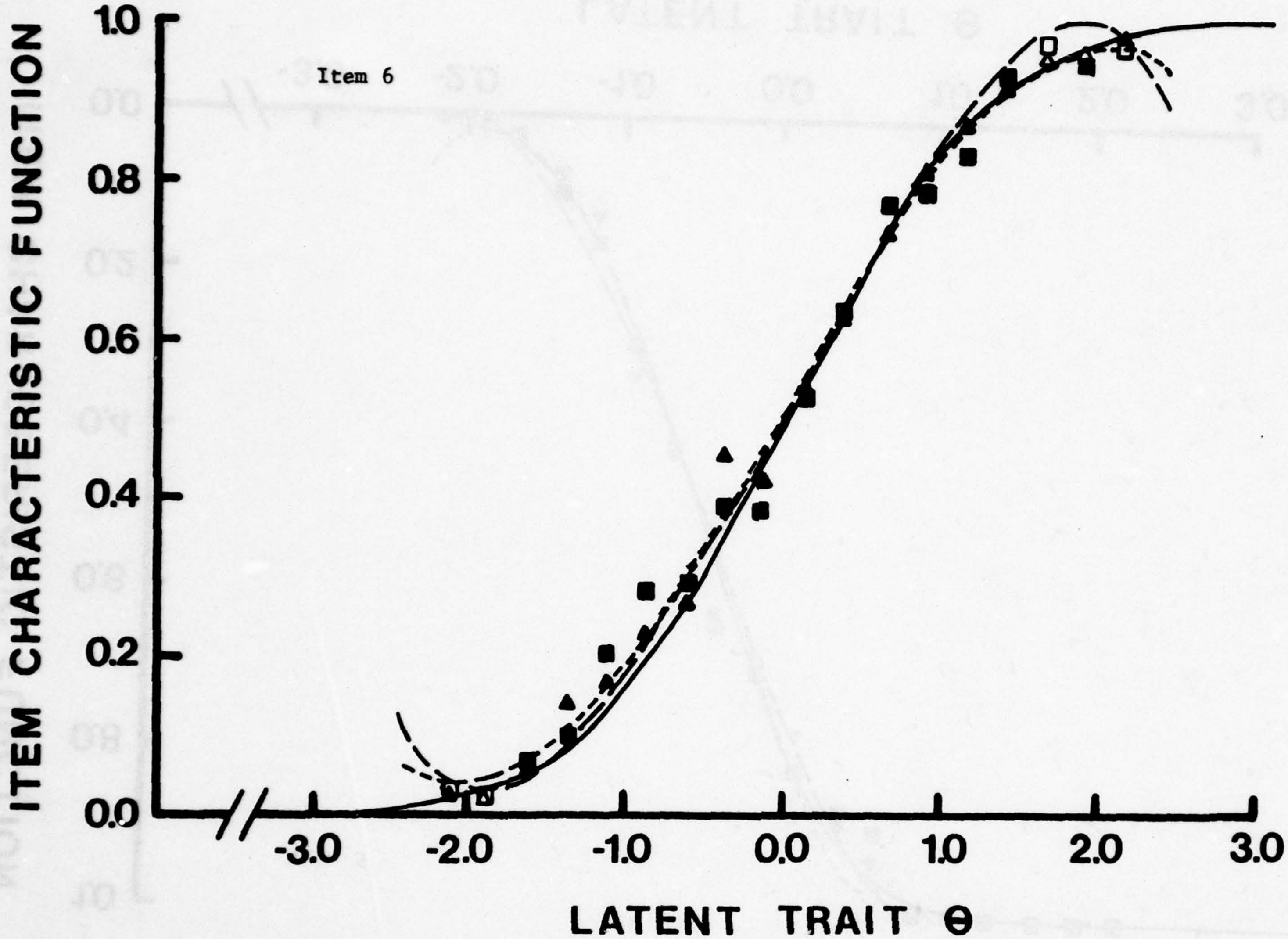

FIGURE 3-3 (Continued)

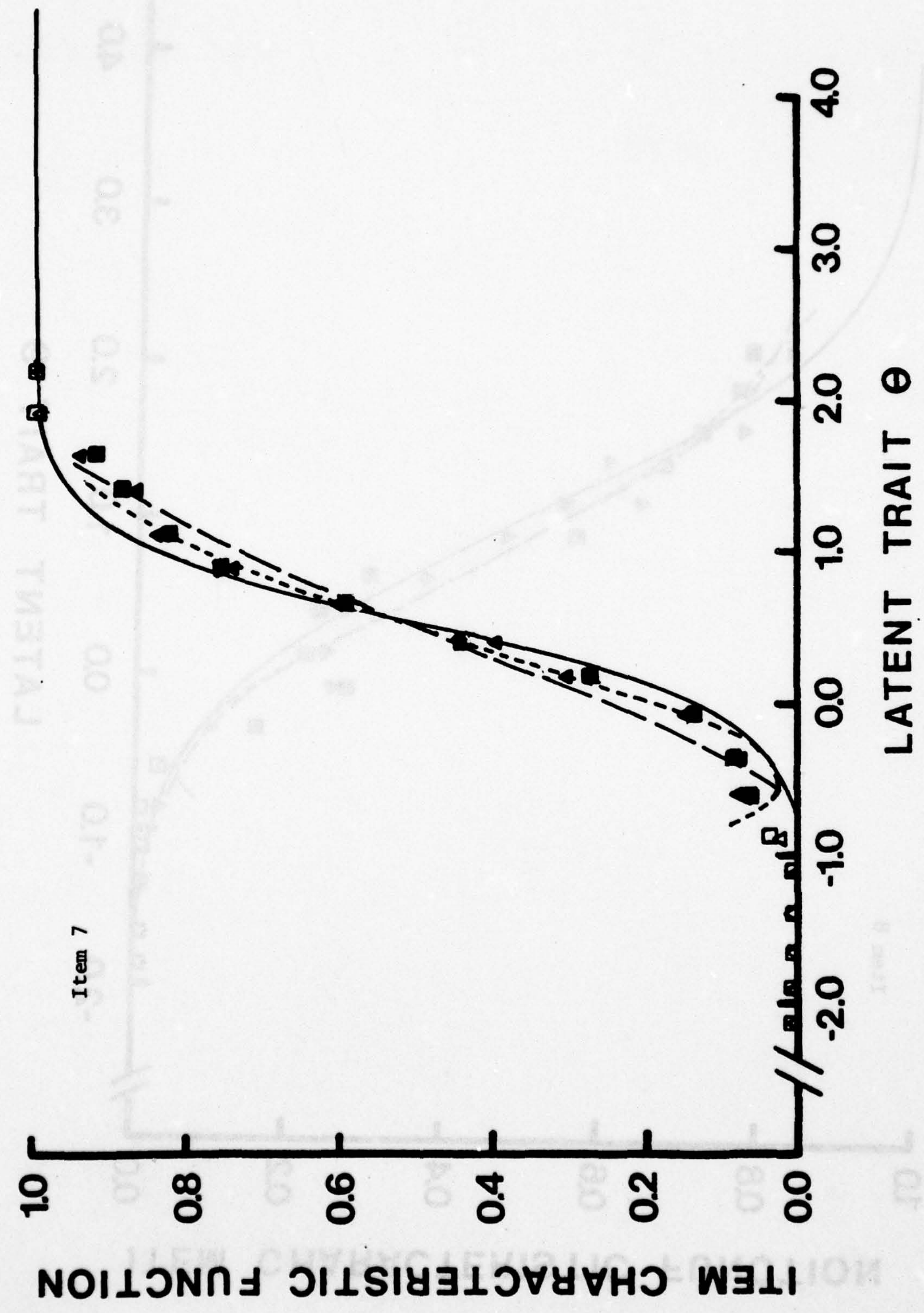

FIGURE 3-3 (Continued)

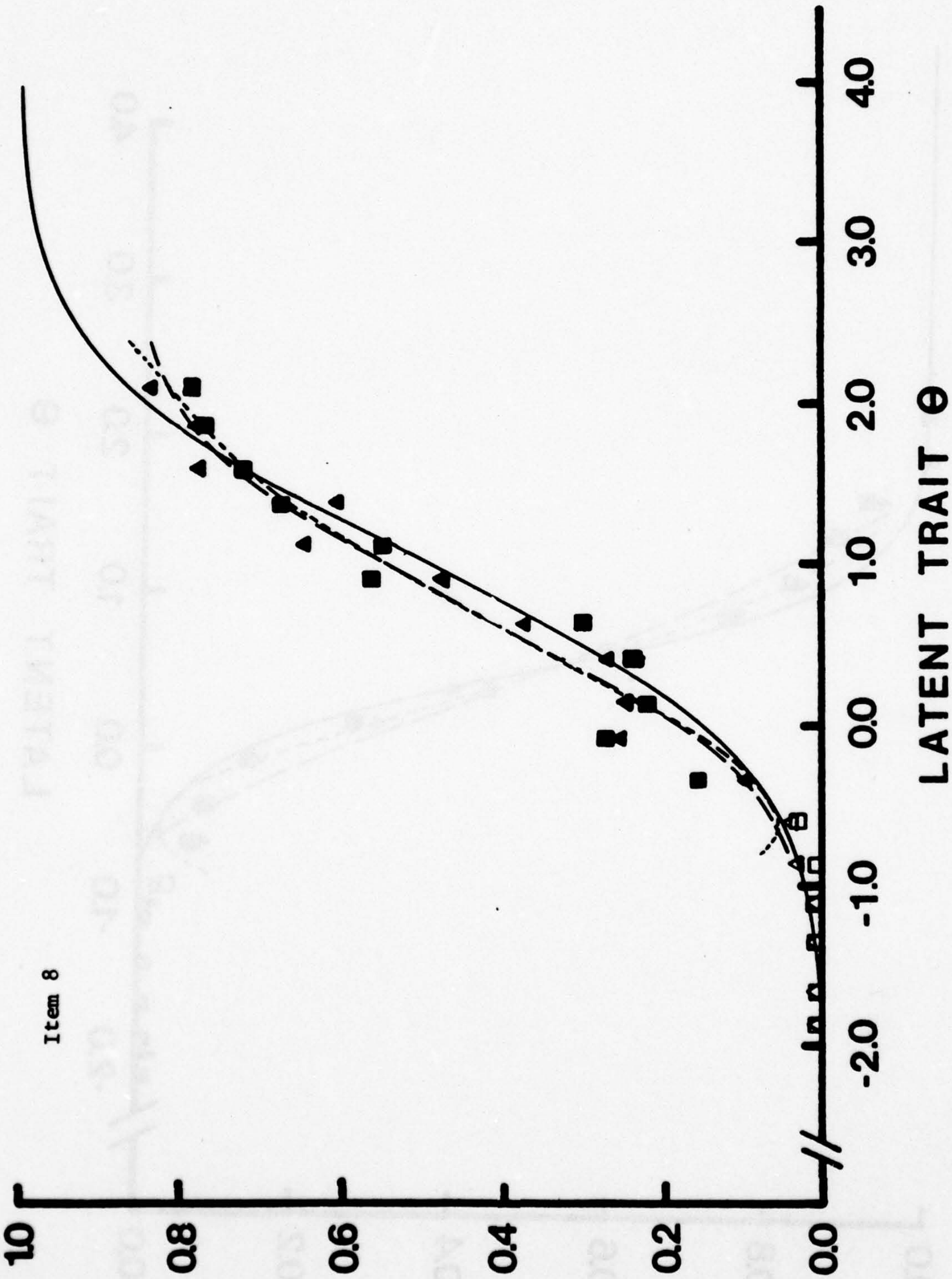

FIGURE 3-3 (Continued)

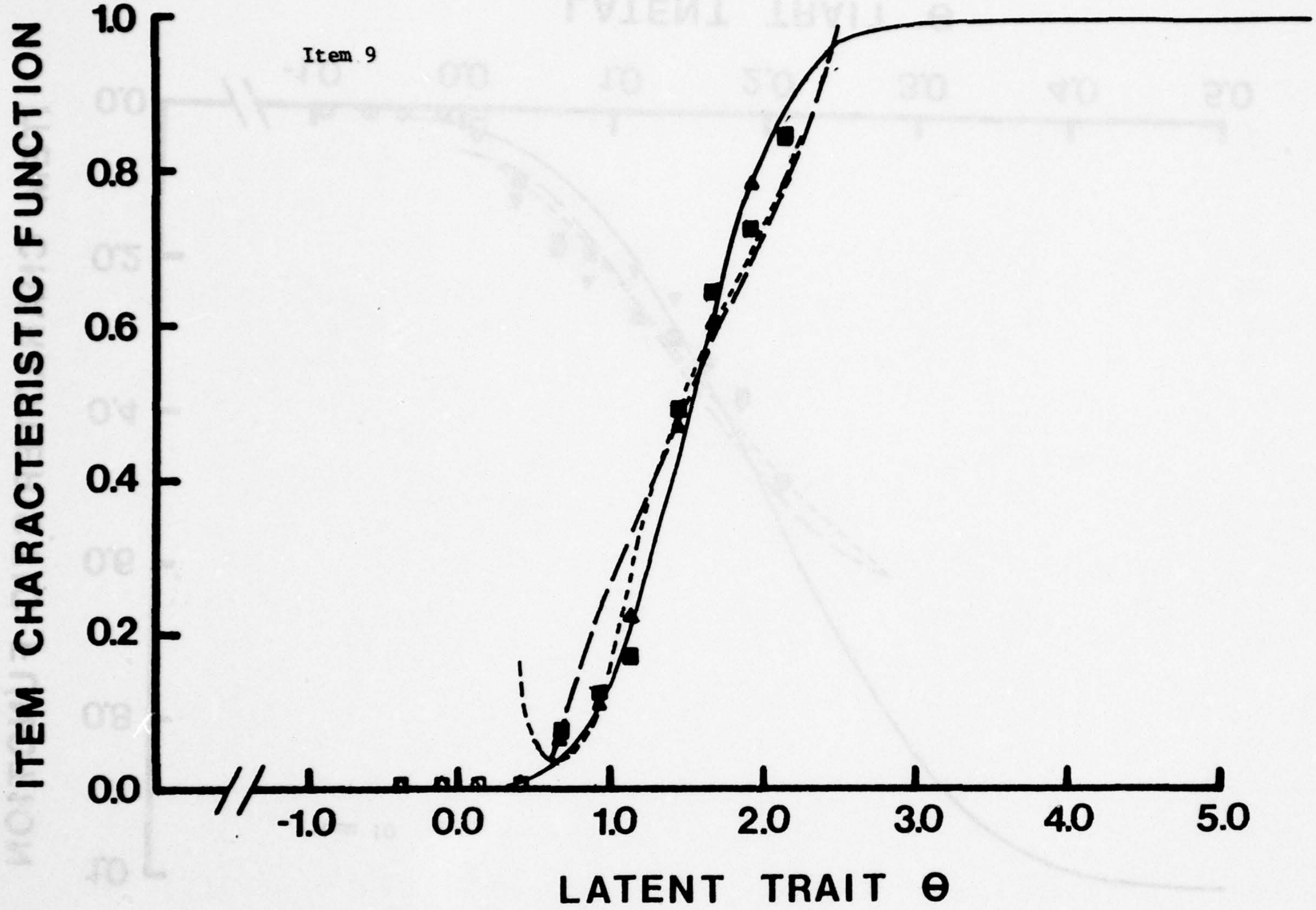

FIGURE 3-3 (Continued)

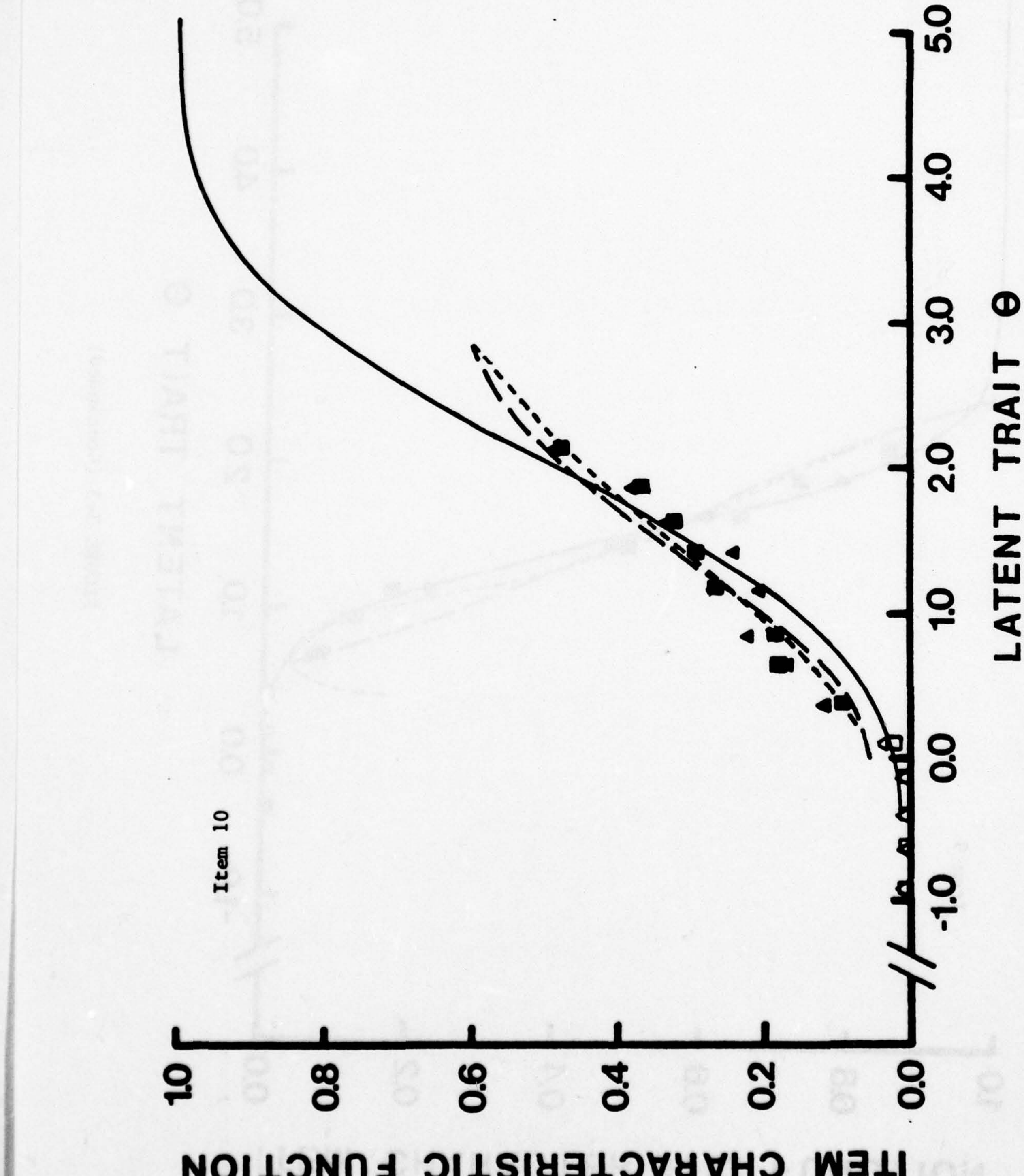

FIGURE 3-3 (Continued)

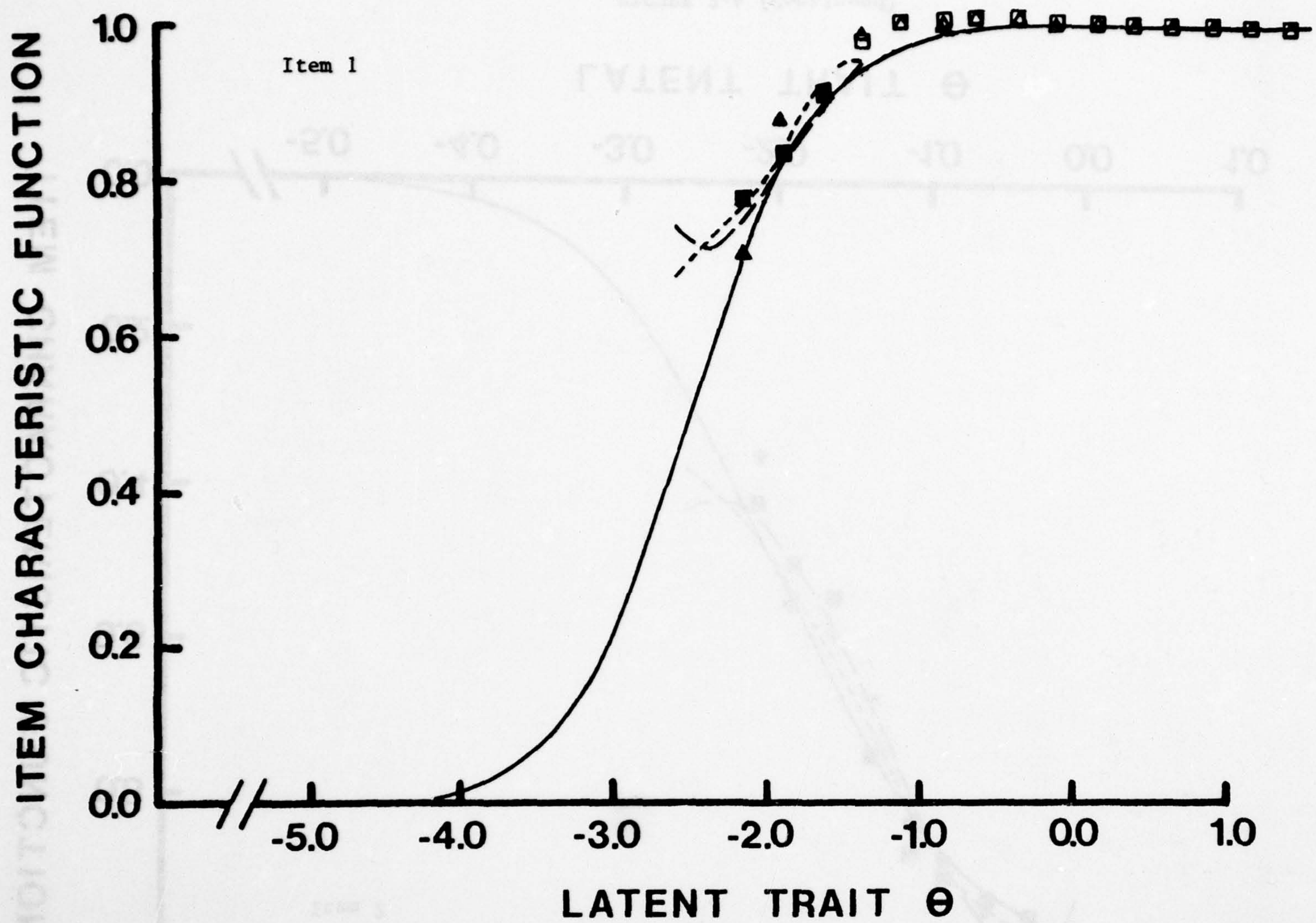

FIGURE 3-4

Estimated Item Characteristic Functions by the Curve Fitting Method for $\tilde{\theta}$, in Degree 4-3 Case (Broken Curve) and Degree 4-4 Case (Dashed Curve), with Those Obtained by the Histogram Ratio Method for $\tilde{\theta}$ in Degree 3 Case (Triangles) and Degree 4 Case (Squares) and the True Item Characteristic Function (Solid Curve)

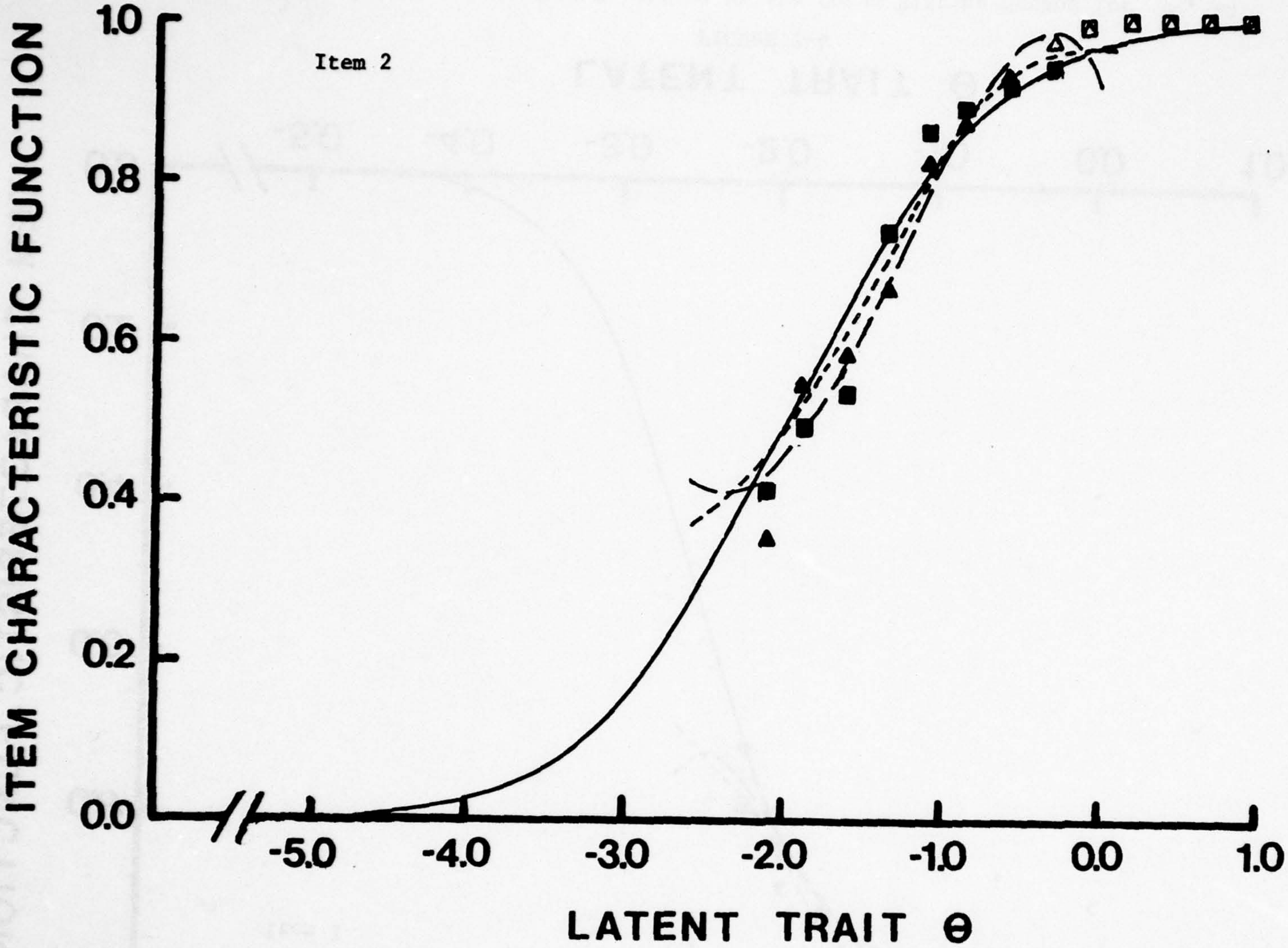

FIGURE 3-4 (Continued)

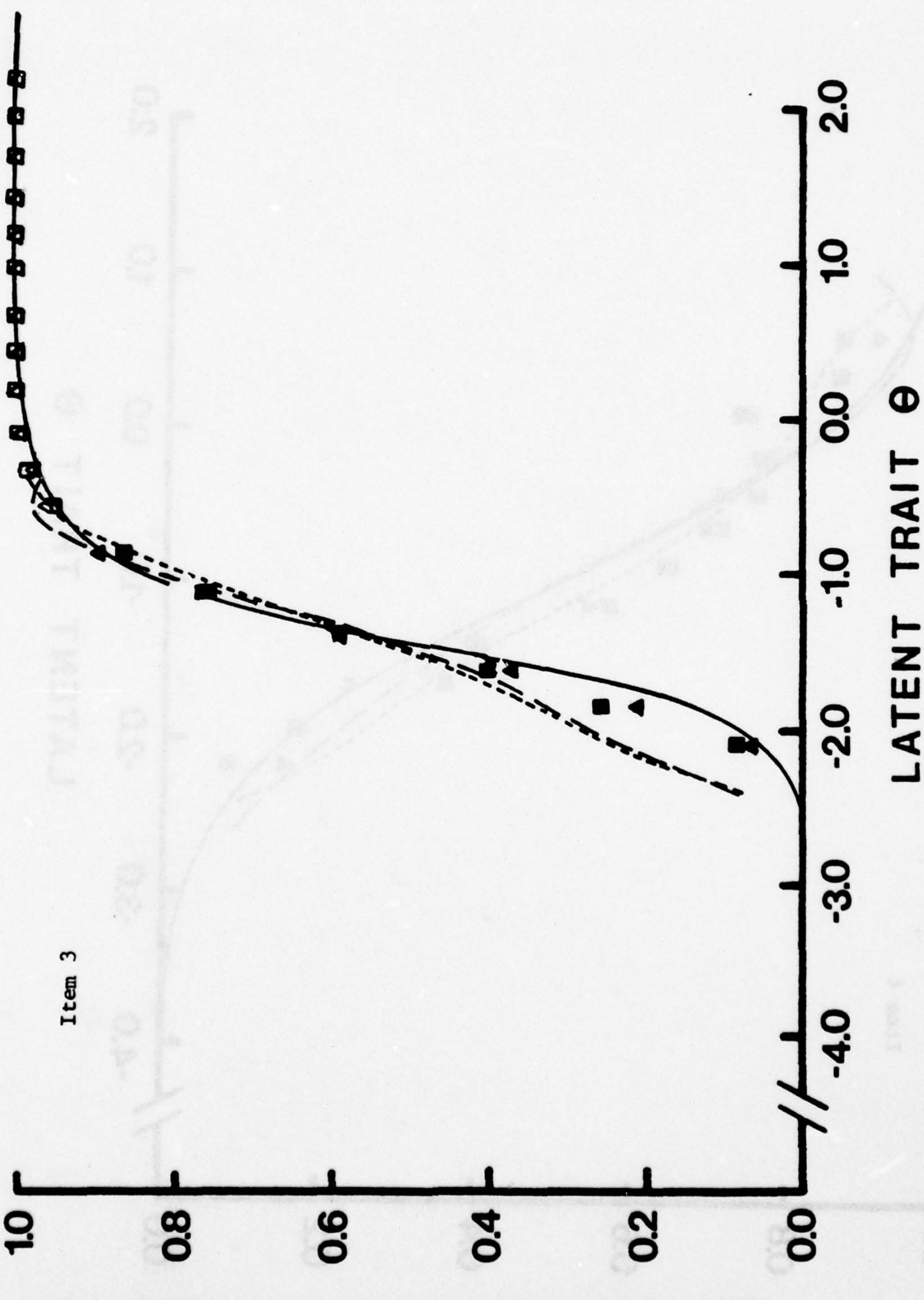

FIGURE 3-4 (Continued)

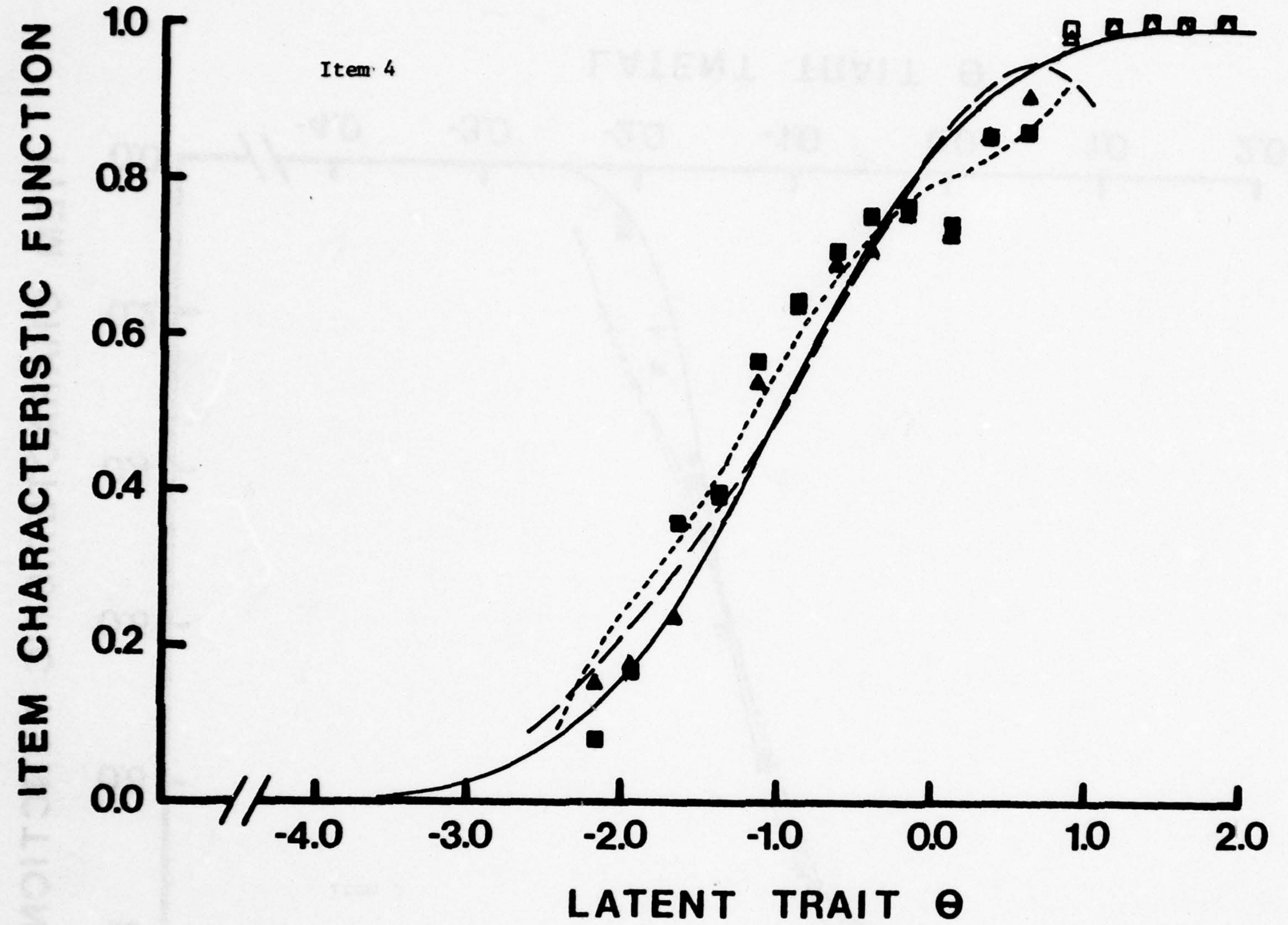

FIGURE 3-4 (Continued)

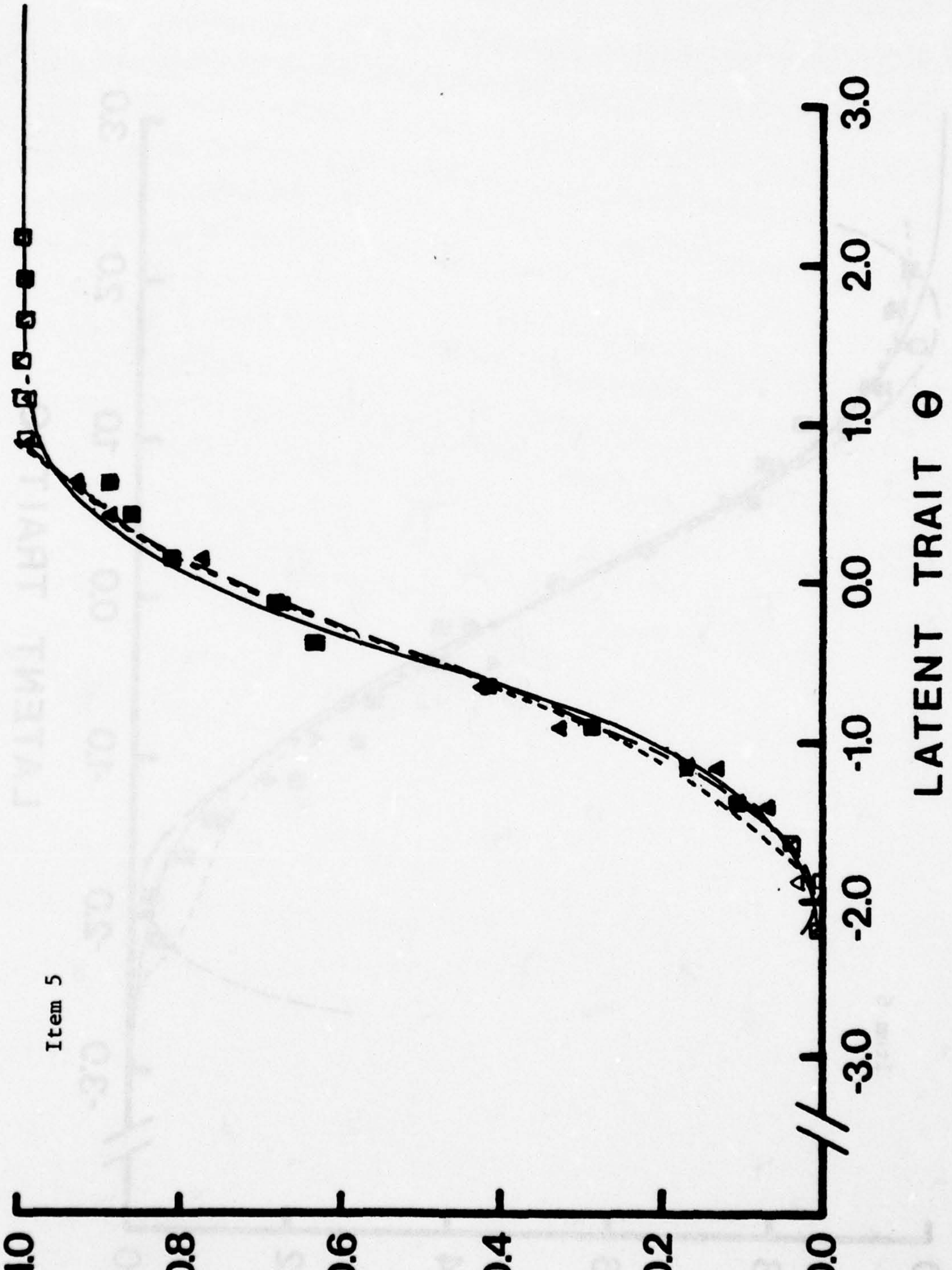

FIGURE 3-4 (Continued)

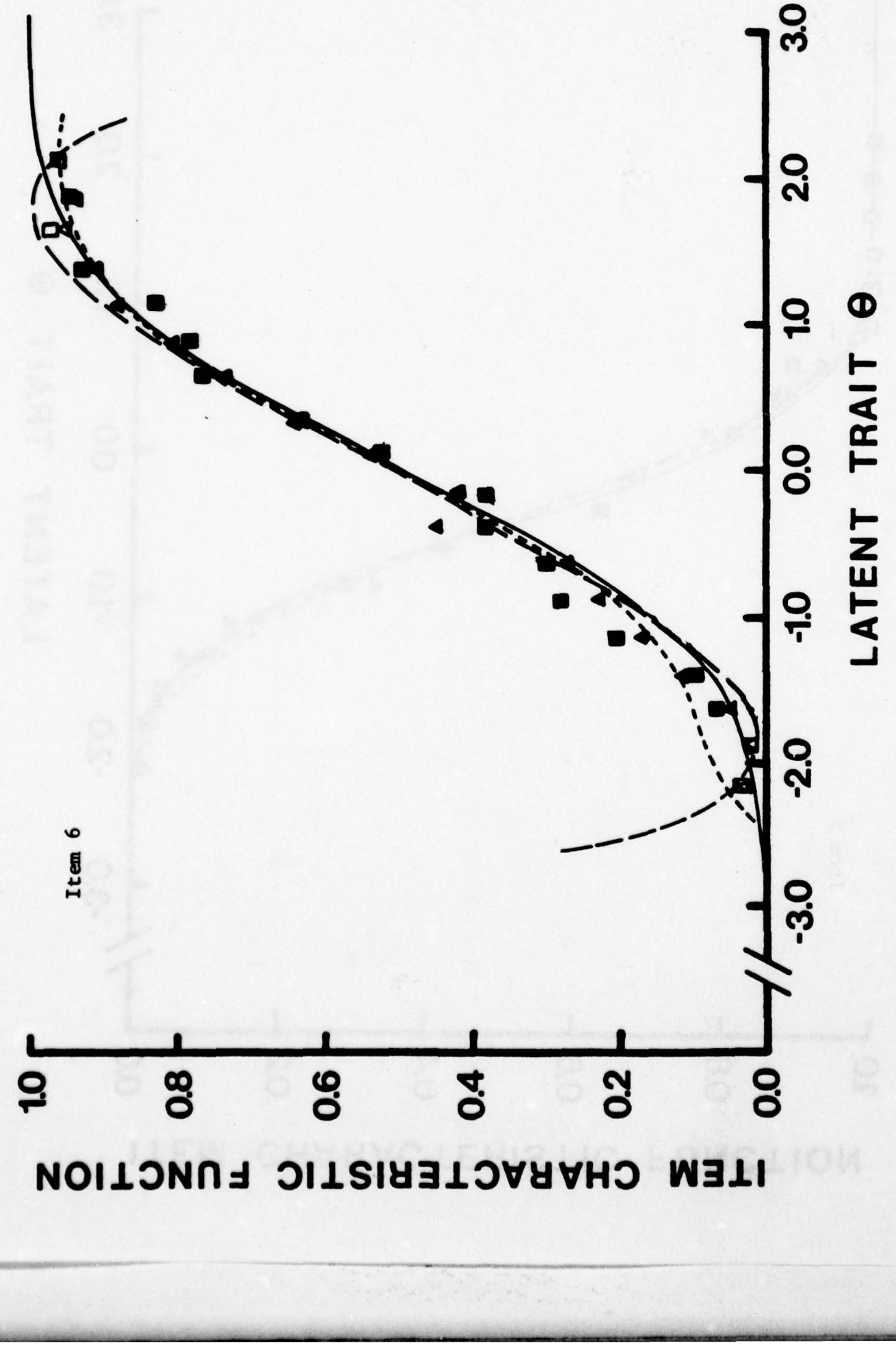

FIGURE 3-4 (Continued)

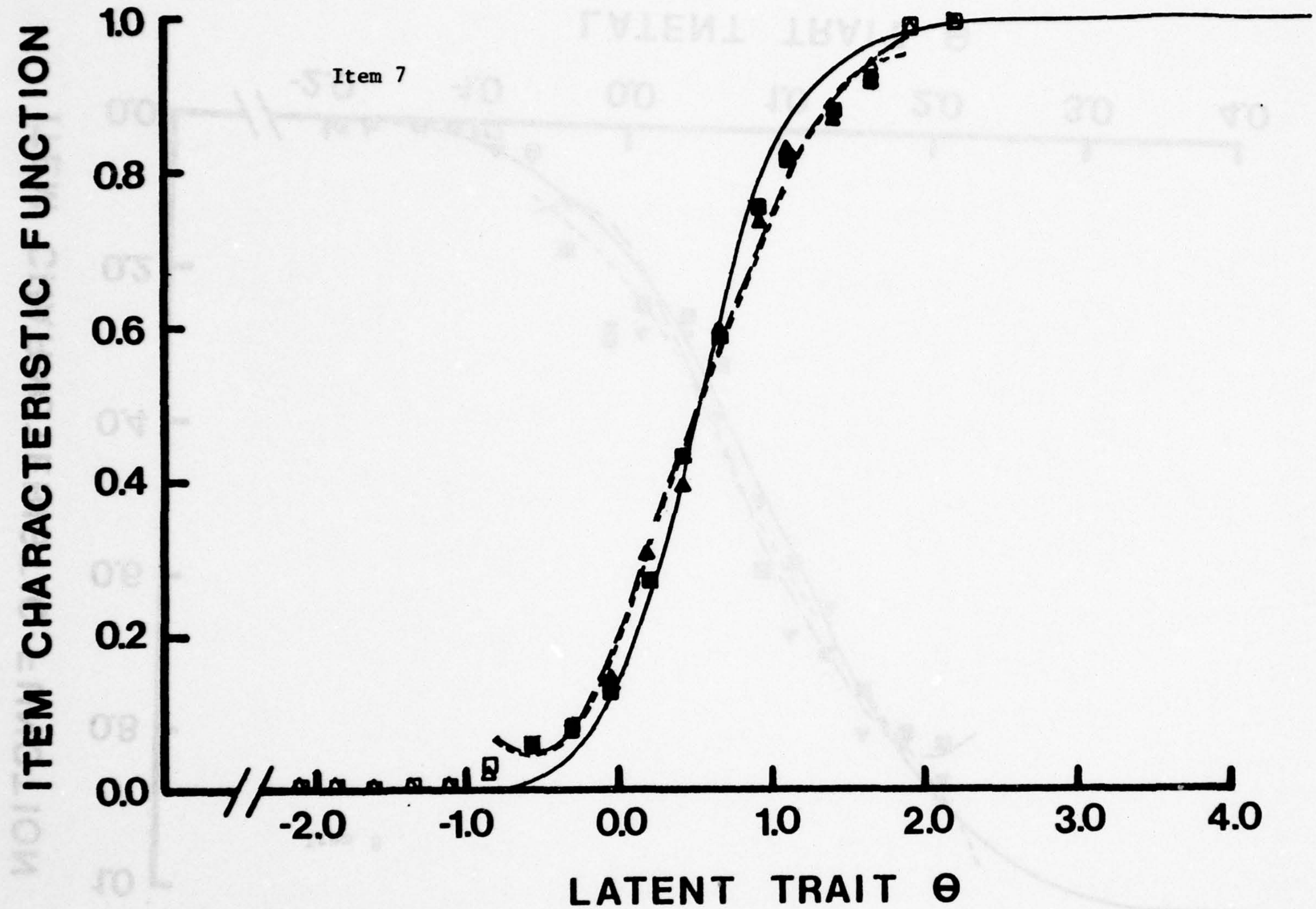

FIGURE 3-4 (Continued)

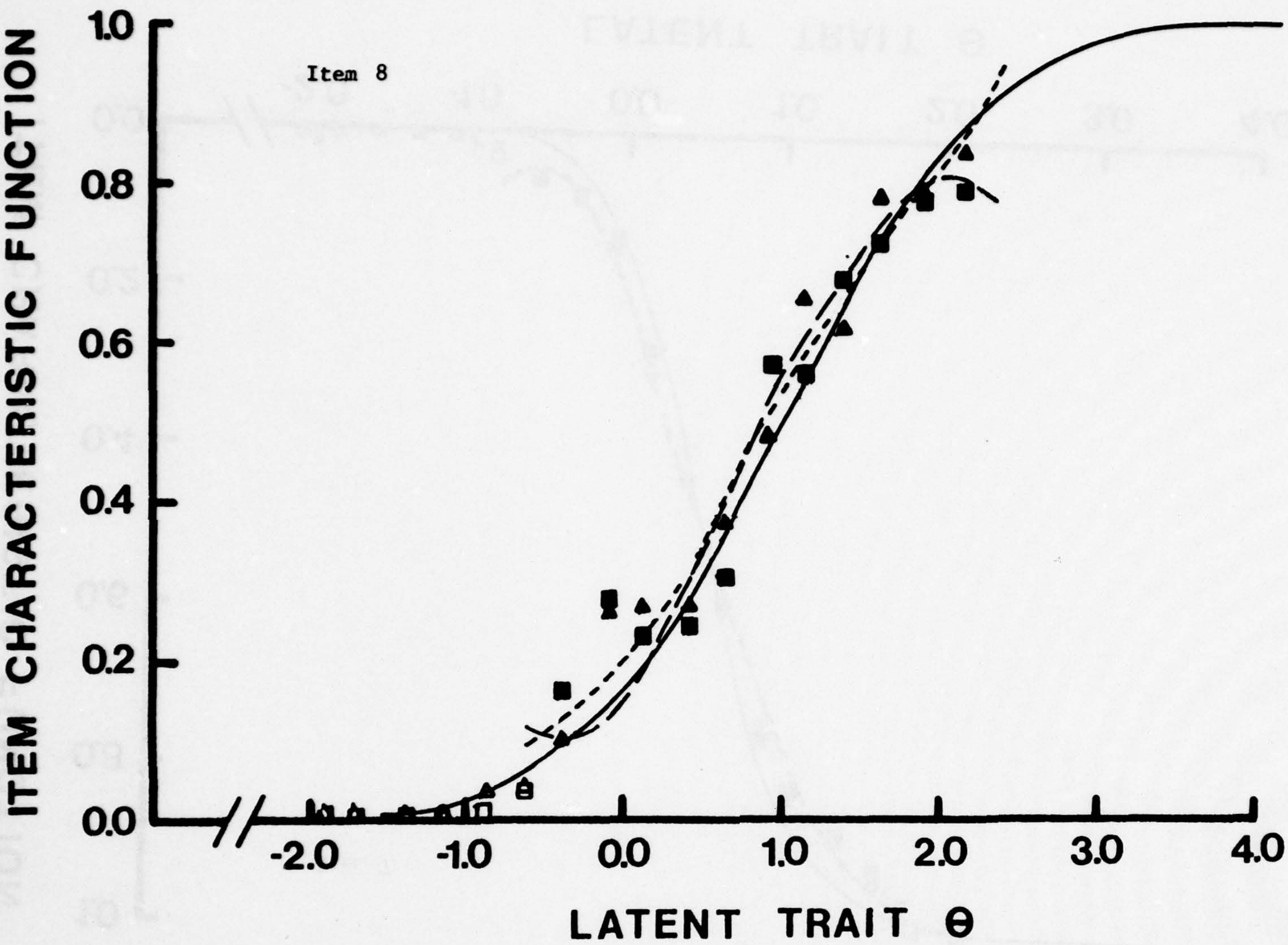

FIGURE 3-4 (Continued)

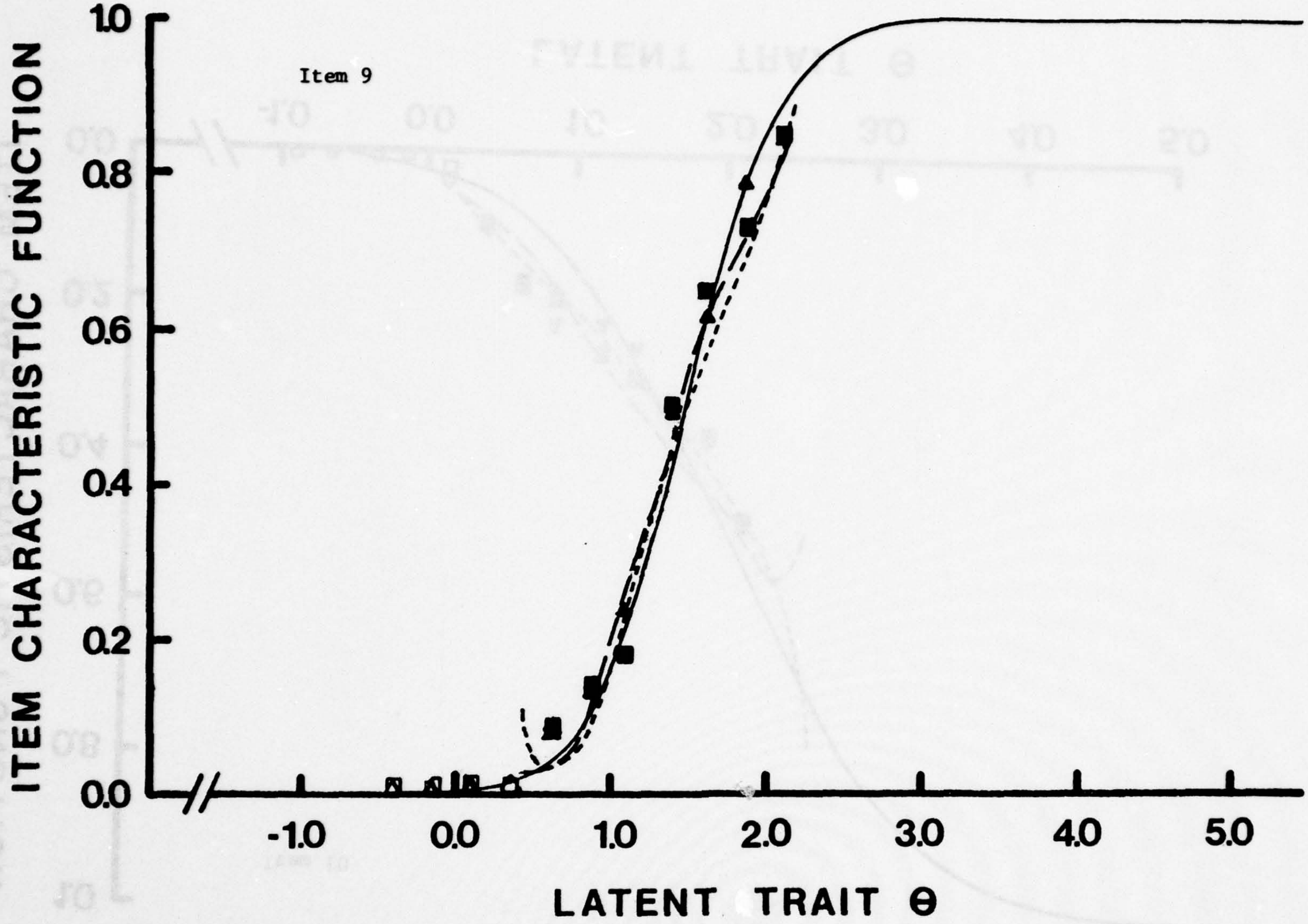

FIGURE 3-4 (Continued)

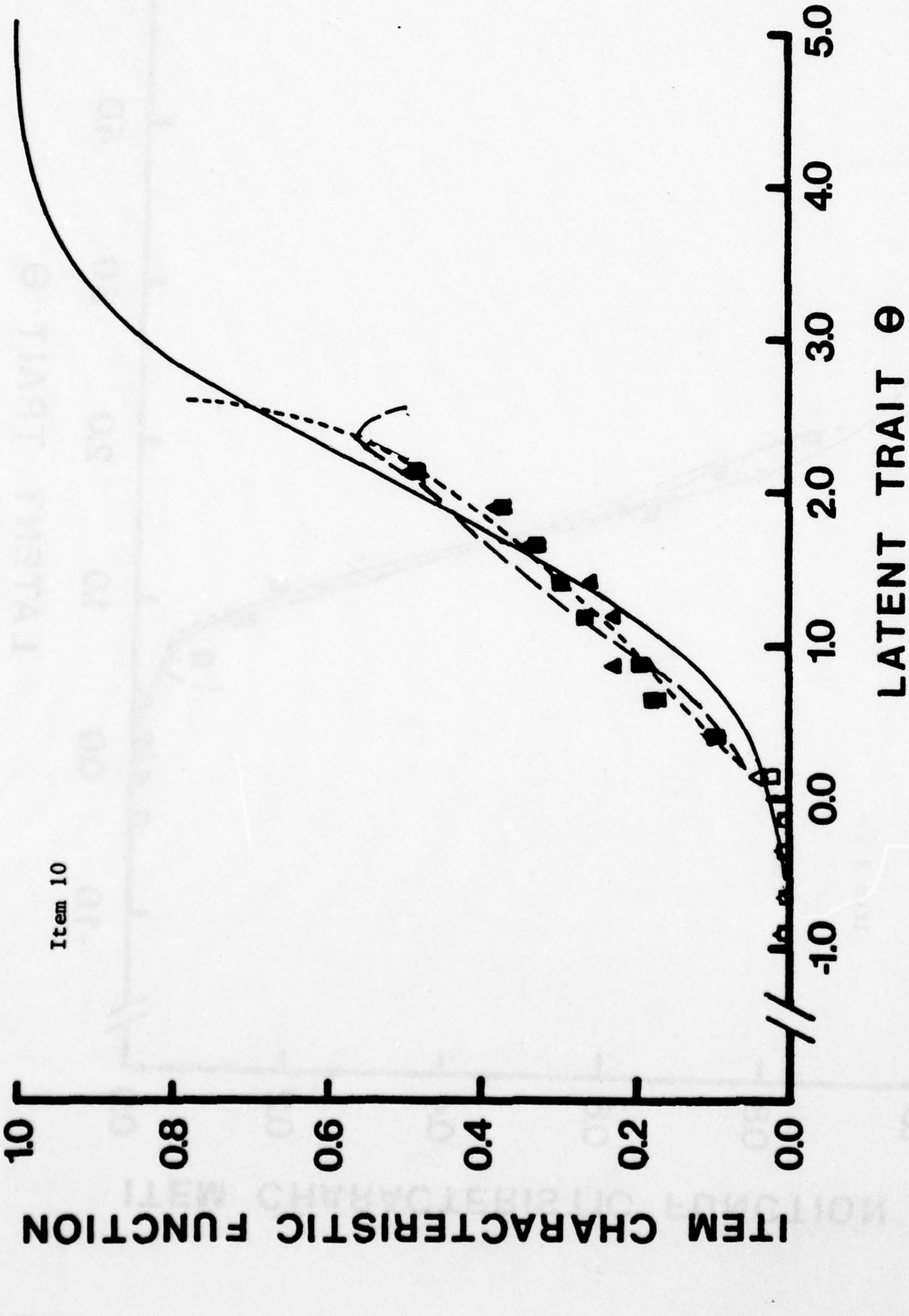

FIGURE 3-4 (Continued)

TABLE 3-3

Discrimination Parameter and Its Estimates of the Ten Binary Items Obtained by the Curve-Fitting Method for $\tilde{\theta}$, i.e., Degree 3-3, 3-4, 4-3 and 4-4 Cases, for θ (3 Cases) and for the Maximum Likelihood Estimates $\hat{\theta}$ (3 Cases); The Range of the Frequency Ratios Used Is [0.05, 0.95], within the Interval of θ, [-2.0, 2.0].

ITEM \ METHOD	TRUE a_g	DGR. 3 θ	DGR. 4 θ	DGR. 5 θ	DGR. 3 MLE	DGR. 4 MLE	DGR. 5 MLE	BETA DGR. 3-3	BETA DGR. 3-4	BETA DGR. 4-3	BETA DGR. 4-4
1	1.5	0.968_3	-----_1	1.536_2	1.710_3	2.175_2	2.889_2	1.201_4	1.315_3	1.319_3	1.845_3
2	1.0	1.223	1.302	1.298	1.147	1.120	1.133	1.062	1.041	0.856	1.136
3	2.5	2.036	2.040	2.095	1.683	1.696	1.891	1.407	1.671	1.649	1.519
4	1.0	0.783	0.804	0.836	0.865	0.742	0.775	0.871	0.795	0.859	0.707
5	1.5	1.293	1.315	1.263	1.303	1.217	1.289	1.286	1.416	1.366	1.289
6	1.0	0.893	0.913	0.922	0.980	0.894	0.909	0.992	0.938	1.064	0.872
7	2.0	1.484	1.548	1.616	1.453	1.403	1.357	1.347	1.383	1.411	1.423
8	1.0	0.888	0.944	0.902	0.801	0.795	0.797	0.959	0.925	0.924	0.864
9	2.0	1.773	1.415	1.555	1.589	1.923	1.981	1.259	1.314	1.583	1.416
10	1.0	0.725	0.748	0.760	0.637	0.538	0.527	0.821	0.749	0.847	0.730

The number of intervals used in estimation is shown as a subscript when it is less than 6.

TABLE 3-4

Difficulty Parameter and Its Estimates of the Ten Binary Items Obtained by the Curve-Fitting Method for $\tilde{\theta}$, i.e., Degree 3-3, 3-4, 4-3 and 4-4 Cases, for θ (3 Cases) and for the Maximum Likelihood Estimates $\hat{\theta}$ (3 Cases); The Range of the Frequency Ratios Used Is [0.05, 0.95], within the Interval of θ, [-2.0, 2.0].

ITEM \ METHOD	TRUE b_g	DGR. 3 θ	DGR. 4 θ	DGR. 5 θ	DGR. 3 MLE	DGR. 4 MLE	DGR. 5 MLE	BETA DGR. 3-3	BETA DGR. 3-4	BETA DGR. 4-3	BETA DGR. 4-4
1	-2.5	-3.231_3	$-----_1$	-2.470_2	-2.419_3	-2.385_2	-2.251_2	-2.671_4	-2.638_3	-2.627_3	-2.461_3
2	-2.0	-1.901	-1.841	-1.803	-1.876	-1.916	-1.908	-1.876	-1.895	-1.955	-1.911
3	-1.5	-1.551	-1.536	-1.508	-1.544	-1.565	-1.493	-1.531	-1.544	-1.541	-1.563
4	-1.0	-1.120	-1.091	-1.081	-1.042	-1.102	-1.053	-1.064	-1.091	-1.060	-1.130
5	-0.5	-0.465	-0.466	-0.461	-0.445	-0.466	-0.452	-0.490	-0.481	-0.473	-0.493
6	0.0	-0.055	-0.051	-0.049	-0.053	-0.090	-0.064	-0.043	-0.057	-0.033	-0.090
7	0.5	0.554	0.520	0.527	0.523	0.532	0.541	0.502	0.461	0.507	0.501
8	1.0	0.990	0.985	0.999	0.982	0.987	0.977	0.950	0.964	0.939	0.971
9	1.5	1.523	1.498	1.496	1.508	1.541	1.526	1.483	1.509	1.496	1.521
10	2.0	2.207	2.163	2.170	2.138	2.372	2.399	2.006	2.100	1.974	2.152

The number of intervals used in estimation is shown as a subscript when it is less than 6.

TABLE 3-5

Discrimination Parameter and Its Estimates of the Ten Binary Items Obtained by the Curve-Fitting Method for $\tilde{\theta}$, i.e., Degree 3-3, 3-4, 4-3 and 4-4 Cases, for θ (3 Cases) and for the Maximum Likelihood Estimates $\hat{\theta}$ (3 Cases); The Range of the Frequency Ratios Used Is [0.01, 0.99], within the Interval of θ , [-2.0, 2.0]

ITEM \ METHOD	TRUE a_g	DGR. 3 θ	DGR. 4 θ	DGR. 5 θ	DGR. 3 MLE	DGR. 4 MLE	DGR. 5 MLE	BETA DGR. 3-3	BETA DGR. 3-4	BETA DGR. 4-3	BETA DGR. 4-4
1	1.5	0.968_3	2.777_2	1.536_2	1.710_3	2.109_3	3.644_3	1.718_5	1.527_5	1.408_4	1.403_4
2	1.0	1.337	1.475	1.409	1.225	1.120	1.133	1.276	1.288	1.073	1.082
3	2.5	2.036	2.040	2.095	1.683	1.696	1.891	1.407	1.671	1.802	1.737
4	1.0	0.783	0.804	0.856	0.865	0.742	0.775	0.875	0.824	0.859	0.707
5	1.5	1.293	1.315	1.263	1.381	1.298	1.360	1.455	1.491	1.423	1.322
6	1.0	0.905	0.963	0.910	1.065	0.894	0.928	1.061	0.936	1.130	0.868
7	2.0	1.742	1.638	1.616	1.527	1.403	1.357	1.490	1.529	1.458	1.420
8	1.0	0.888	0.944	0.902	0.801	0.795	0.797	0.962	0.925	0.924	0.864
9	2.0	1.773	1.415	1.730	1.589	1.923	1.981	1.519	1.469	1.658	1.595
10	1.0	0.725	0.748	0.760	0.637	0.538	0.527	0.821	0.794	0.847	0.730

The number of intervals used in estimation is shown as a subscript when it is less than 6.

TABLE 3-6

Difficulty Parameter and Its Estimates of the Ten Binary Items Obtained by the Curve-Fitting Method for $\tilde{\theta}$, i.e., Degree 3-3, 3-4, 4-3 and 4-4 Cases, for θ (3 Cases) and for the Maximum Likelihood Estimates $\hat{\theta}$ (3 Cases); The Range of the Frequency Ratios Used Is [0.01, 0.99], within the Interval of θ, [-2.0, 2.0].

ITEM \ METHOD	TRUE b_g	DGR. 3 θ	DGR. 4 θ	DGR. 5 θ	DGR. 3 MLE	DGR. 4 MLE	DGR. 5 MLE	BETA DGR. 3-3	BETA DGR. 3-4	BETA DGR. 4-3	BETA DGR. 4-4
1	-2.5	-3.231_3	-2.253_2	-2.470_2	-2.419_3	-2.399_3	-2.185_3	-2.409_5	-2.535_5	-2.579_4	-2.648_4
2	-2.0	-1.886	-1.827	-1.819	-1.862	-1.916	-1.908	-1.827	-1.836	-1.900	-1.936
3	-1.5	-1.551	-1.536	-1.508	-1.544	-1.565	-1.493	-1.531	-1.544	-1.550	-1.565
4	-1.0	-1.120	-1.091	-1.083	-1.042	-1.102	-1.053	-1.064	-1.090	-1.060	-1.130
5	-0.5	-0.465	-0.466	-0.461	-0.435	-0.449	-0.438	-0.512	-0.480	-0.460	-0.484
6	0.0	-0.064	-0.068	-0.040	-0.109	-0.091	-0.080	-0.080	-0.062	-0.029	-0.087
7	0.5	0.492	0.541	0.527	0.501	0.532	0.541	0.513	0.504	0.504	0.515
8	1.0	0.990	0.985	0.999	0.982	0.987	0.977	0.950	0.964	0.939	0.971
9	1.5	1.523	1.498	1.519	1.508	1.541	1.526	1.503	1.523	1.504	1.535
10	2.0	2.207	2.163	2.170	2.138	2.372	2.399	2.006	2.061	1.974	2.152

The number of intervals used in estimation is shown as a subscript when it is less than 6.

TABLE 3-7

Discrimination Parameter and Its Estimates of the Ten Binary Items Obtained by the Curve-Fitting Method for $\tilde{\theta}$, i.e., Degree 3-3, 3-4, 4-3 and 4-4 Cases, for θ (3 Cases) and for the Maximum Likelihood Estimates $\hat{\theta}$ (3 Cases); The Range of the Frequency Ratios Used Is [0.05, 0.95], within the Interval of θ, [-3.0, 3.0].

ITEM \ METHOD	TRUE a_g	DGR. 3 θ	DGR. 4 θ	DGR. 5 θ	DGR. 3 MLE	DGR. 4 MLE	DGR. 5 MLE	BETA DGR. 3-3	BETA DGR. 3-4	BETA DGR. 4-3	BETA DGR. 4-4
1	1.5	2.042_5	1.868_3	1.661_4	0.735	1.103_5	0.840_5	0.799	0.872	0.965_5	1.249_5
2	1.0	1.095	1.052	1.013	0.919	0.914	1.010	0.533	0.550	0.822	1.016
3	2.5	2.036	2.040	2.095	1.626	1.646	1.549	1.480	1.631	1.579	1.539
4	1.0	0.841	0.851	0.886	0.852	0.800	0.846	0.881	0.824	0.859	0.761
5	1.5	1.293	1.315	1.263	1.303	1.217	1.289	1.286	1.220	1.366	1.289
6	1.0	0.893	0.913	0.922	0.731	0.834	0.815	0.711	0.827	0.740	0.853
7	2.0	1.484	1.548	1.616	1.453	1.403	1.357	1.347	1.383	1.411	1.423
8	1.0	0.933	0.942	0.921	0.801	0.795	0.797	0.887	0.891	0.832	0.902
9	2.0	1.773	1.627	1.586	1.562	1.923	1.928	1.280	1.340	1.557	1.526
10	1.0	0.810	0.804	0.828	0.562	0.619	0.628	0.716	0.660	0.752	0.716

The number of intervals used in estimation is shown as a subscript when it is less than 6.

TABLE 3-8

Difficulty Parameter and Its Estimates of the Ten Binary Items Obtained by the Curve-Fitting Method for $\tilde{\theta}$, i.e., Degree 3-3, 3-4, 4-3 and 4-4 Cases, for θ (3 Cases) and for the Maximum Likelihood Estimates $\hat{\theta}$ (3 Cases); The Range of the Frequency Ratios Used Is [0.05, 0.95], within the Interval of θ , [-3.0, 3.0].

ITEM \ METHOD	TRUE b_g	DGR. 3 θ	DGR. 4 θ	DGR. 5 θ	DGR. 3 MLE	DGR. 4 MLE	DGR. 5 MLE	BETA DGR. 3-3	BETA DGR. 3-4	BETA DGR. 4-3	BETA DGR. 4-4
1	-2.5	-2.456_5	-2.434_3	-2.430_4	-3.257	-2.838_5	-3.070_5	-3.161	-3.081	-2.942_5	-2.793_5
2	-2.0	-1.986	-2.000	-1.995	-2.069	-2.117	-2.014	-2.415	-2.403	-1.996	-2.006
3	-1.5	-1.551	-1.536	-1.508	-1.556	-1.576	-1.526	-1.520	-1.557	-1.556	-1.561
4	-1.0	-1.044	-1.031	-1.026	-1.053	-1.045	-0.988	-1.055	-1.059	-1.060	-1.062
5	-0.5	-0.465	-0.466	-0.461	-0.445	-0.466	-0.452	-0.490	-0.536	-0.473	-0.493
6	0.0	-0.055	-0.051	-0.049	0.024	-0.049	-0.019	-0.021	-0.050	-0.002	-0.101
7	0.5	0.554	0.520	0.527	0.523	0.532	0.541	0.502	0.461	0.507	0.501
8	1.0	0.955	0.987	0.984	0.982	0.987	0.977	1.001	0.994	1.009	0.944
9	1.5	1.523	1.450	1.489	1.513	1.541	1.535	1.479	1.499	1.501	1.484
10	2.0	2.062	2.069	2.060	2.269	2.195	2.176	2.186	2.262	2.117	2.178

The number of intervals used in estimation is shown as a subscript when it is less than 6.

IV Conditional P.D.F. Method

One of the reasons why $\tilde{\theta}$ was calibrated by the Monte Carlo method to multiply the data in the preceding study (Samejima, 1977d) is to compare the results with those obtained by the Normal Approximation Method. Another reason is to compare them directly with the results obtained from the set of five hundred true ability values themselves, such as the frequency ratio of the examinees who answered item g correctly to the total group of examinees for each of the twenty subintervals of θ , etc., which are not observable in the empirical situation. In fact, the frequency ratios obtained by the Two-Parameter Beta Method turned out to give closer estimates of the true item characteristic functions than those obtained directly from the set of five hundred true ability scores, and the estimated parameter values in the normal ogive model obtained by the Two-Parameter Beta Method are as good as those obtained directly from the ability scores (cf. Samejima, 1977d, Figures 7-9 and 8-3, Tables 7-1, 7-2, 8-1 and 8-2).

It is a logical speculation, therefore, that the use of the total conditional probability density function of θ , given $\hat{\theta}$, instead of the calibrated $\tilde{\theta}$'s , may still increase the accuracy of the estimation of the operating characteristics. For this reason, the second half of the present study is focused upon investigating this possibility, as well as finding out its limitations.

We follow the same procedure up to the fourth step described in Section II, and obtain $\hat{\phi}(\theta|\hat{\theta}_s)$, the conditional density function of θ , given $\hat{\theta}$, which is estimated by the Two-Parameter Beta Method.

The estimated item characteristic function, $\hat{P}_g(\theta)$, is given by

(4.1) $$\hat{P}_g(\theta) = \sum_{s\varepsilon G} \hat{\phi}(\theta|\hat{\theta}_s)[\sum_{s=1}^{N} \hat{\phi}(\theta|\hat{\theta}_s)]^{-1}$$

where s is the individual examinee, G is the group of examinees who answered item g correctly, N (= 500) is the number of examinees in the total group, and $\hat{\phi}(\theta|\hat{\theta})$ is the approximated conditional density function given by (2.5) . The a priori set parameters, $a_{\hat{\theta}}$ and $b_{\hat{\theta}}$, for the Beta distribution are the same as those used in the previous study (Samejima, 1977d), i.e., $\hat{\theta} \pm 0.54825$; and the other two parameters, $p_{\hat{\theta}}$ and $q_{\hat{\theta}}$, are also the same as before, which were obtained from the conditional expectation and variance of θ , given $\hat{\theta}$, and the estimate of $g(\hat{\theta})$ which were approximated by the polynomial of degree 3 or 4, through equations (2.6) and (2.7).

From the approximated conditional density function, $\hat{\phi}(\theta|\hat{\theta}_s)$, we can obtain an estimate of the probability density function of θ , such that

(4.2) $$\hat{f}(\theta) = \frac{1}{N} \sum_{s=1}^{N} \hat{\phi}(\theta|\hat{\theta}_s) .$$

It is expected that, if this estimated density function is close to the true density function, $f(\theta)$, then the estimated operating characteristics obtained by the method in question will be reasonably close to the true operating characteristics.

V Criterion Item Characteristic Function

As a criterion for evaluating the resulting estimates of the item characteristic function, we will use the counterpart of (4.1) by replacing $\hat{\phi}(\theta|\hat{\theta})$ by the true conditional density of θ, given $\hat{\theta}$, which is given by

$$(5.1)\qquad \phi(\theta|\hat{\theta}) = \psi(\hat{\theta}|\theta)\, f(\theta)\, \left[\int_{-\infty}^{\infty} \psi(\hat{\theta}|\theta)\, f(\theta)\, d\theta\right]^{-1} ,$$

where $\psi(\hat{\theta}|\theta)$ is approximated by the normal density with θ and the inverse of the test characteristic function $I(\theta)$ $(= 21.63)$ as its two parameters, by virtue of the asymptotic property of the maximum likelihood estimate (Samejima, 1975, 1977a, 1977b), and $f(\theta)$ $(= 0.2)$ is the probability density function of θ. Hereafter, this function is called the <u>criterion item characteristic function</u>. Just like the frequency ratios of the true θ, the criterion item characteristic function is not observable in the empirical situation, and only the simulation study can receive its benefit.

Note that this criterion item characteristic function <u>does</u> depend upon the sampling fluctuations of the examinee's item responses to the binary item. In this sense, this criterion itself will show the limitation of the methods, in which (4.1) is used for the estimated item characteristic function, however refined $\phi(\theta|\hat{\theta})$ may be.

The criterion item characteristic function is also beneficial in the sense that it provides us with a continuous function, rather than a discontinuous set of frequency ratios, in the type of research like the present one. It will be fully used not only in the present study, but in the future research on the estimation of the operating characteristics.

In the following section, the criterion item characteristic function for each of the ten binary items is drawn by a thin, solid line to be compared with the resulting estimated item characteristic functions by the Conditional P.D.F. Method of the Two-Parameter Beta Method. The set of frequency ratios obtained from the five hundred true ability scores is also presented in the same figures, using a broken and dotted line.

VI Results

Figures 6-1 and 6-2 present the estimated probability density functions of θ (solid curve) obtained by (4.2), for Degree 3 and 4 Cases respectively. In the same graphs, the theoretical probability density function, $f(\theta)$, (dashed line) and the frequency ratios of $\tilde{\theta}$ with 0.25 as the width of intervals (solid line), which were obtained previously (Samejima, 1977d), are also drawn.

As is expected, these curves are very close to the corresponding histograms of $\tilde{\theta}$, smoothing out its sampling fluctuations caused by the calibration of $\tilde{\theta}$ by the Monte Carlo method. The curves are not so close to the theoretical density $f(\theta)$, however, especially for the extreme values of θ. This is considered, mainly, to be due to the two artificially set parameter values for each Beta density function. We can also see that there is a slight difference between Degree 3 and 4 Cases, i.e., the curve is closer to $f(\theta)$ as a whole in Degree 4 Case than in Degree 3 Case. This may be caused partly because of the better approximation of $g(\hat{\theta})$ in Degree 4 Case; at the same time, however, we should note that in Degree 4 Case seven extreme values of $\hat{\theta}$ were excluded, and this must be another reason for the better fit in Degree 4 Case at the extreme values of θ.

Figure 6-3 presents the estimated item characteristic functions obtained by the Conditional P.D.F. Method of the Two-Parameter Beta Method, in Degree 3 Case (broken curve) and in Degree 4 Case (dashed curve), together with the theoretical item characteristic function (thick solid curve) and the criterion item characteristic function

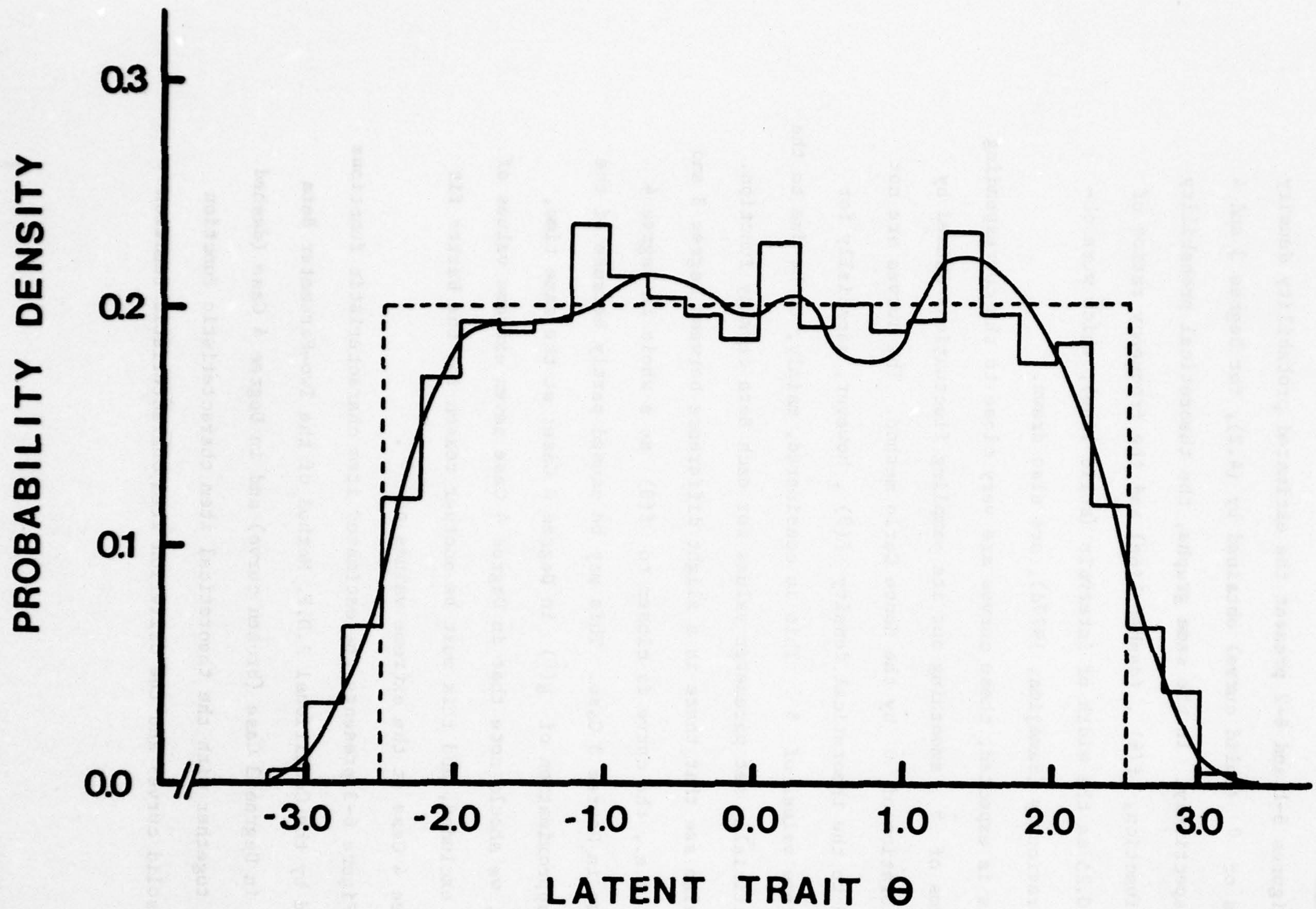

FIGURE 6-1

Approximated Marginal Density Function of θ Obtained from the 500 Conditional Density Functions of θ, Given Its Maximum Likelihood Estimate, Which Is Approximated by a Beta Density Function with Two A Priori Set Parameters and Two Estimated Parameters: Degree 3 Case (Solid Curve), with the Relative Frequency Distribution of $\tilde{\theta}$ (Histogram) and the Theoretical Density Function of θ (Dashed Line)

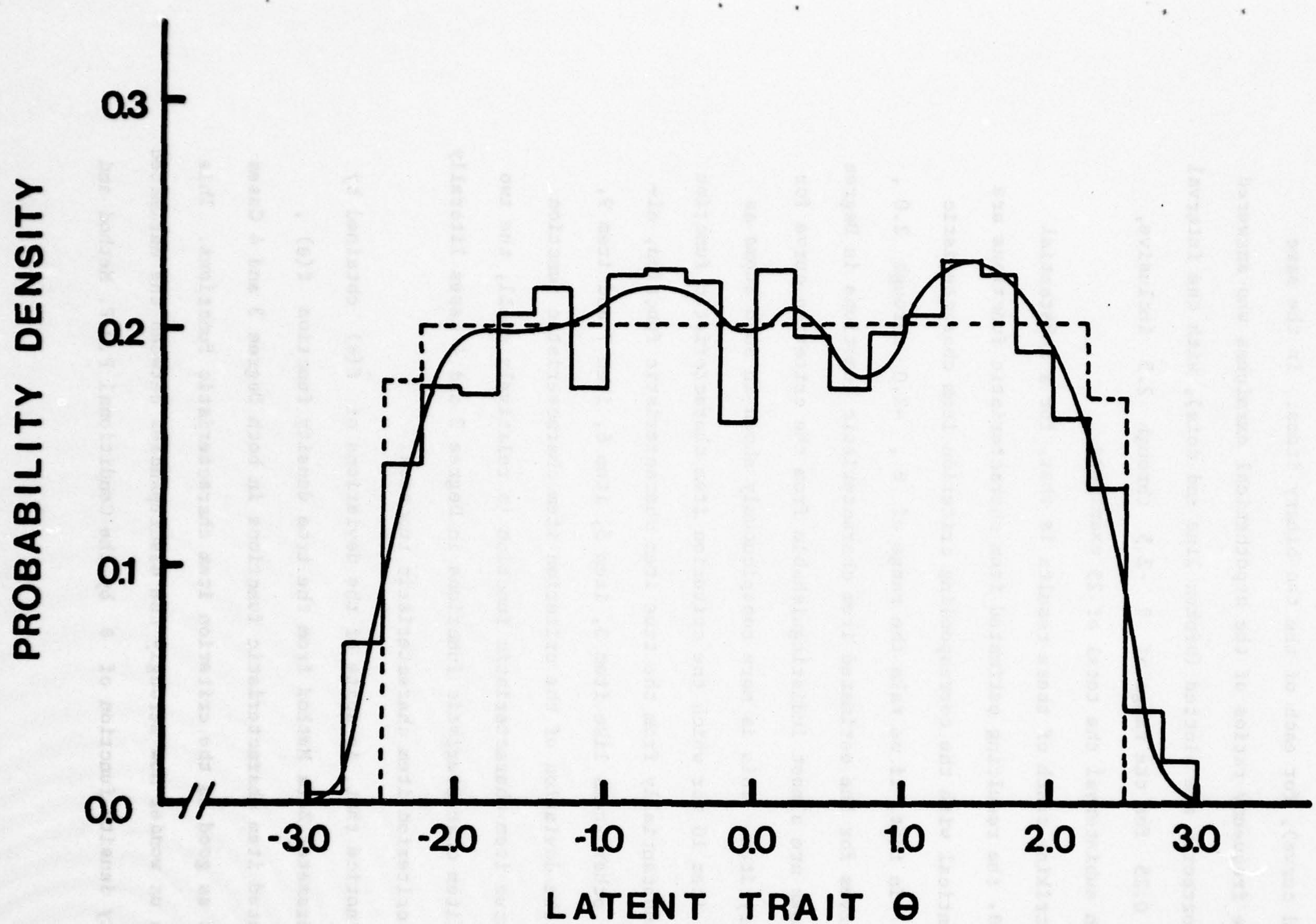

FIGURE 6-2

Approximated Marginal Density Function of θ Obtained from the 500 Conditional Density Functions of θ, Given Its Maximum Likelihood Estimate, Which Is Approximated by a Beta Density Function with Two A Priori Set Parameters and Two Estimated Parameters: Degree 4 Case (Solid Curve), with the Relative Frequency Distribution of $\tilde{\theta}$ (Histogram) and the Theoretical Density Function of θ (Dashed Line)

(thin solid curve), for each of the ten binary items. In the same graphs, the frequency ratios of the hypothetical examinees who answered the item correctly are plotted (broken line and dots), with the interval length of 0.25 for the range of θ -2.5 through 2.5 inclusive, giving each subinterval the total of 25 examinees.

A striking truth of these results is that, for a substantial range of θ, the resulting estimated item characteristic functions are almost identical with the corresponding criterion item characteristic functions. In fact, if we take the range of θ , -2.0 through 2.0 , the two curves for the estimated item characteristic functions in Degree 3 and 4 Cases are almost indistinguishable from the criterion curve for every binary item. This is more conspicuously shown in such items as item 4 and item 10 for which the criterion item characteristic function deviates substantially from the true item characteristic function, although for other items like item 3, item 5, item 6, item 7 and item 9, for which the deviation of the criterion item characteristic function from the true item characteristic function is relatively small, the two estimated item characteristic functions in Degree 3 and 4 Cases literally trace the criterion item characteristic function.

We notice that, in spite of the deviations of $\hat{f}(\theta)$ obtained by the Two-Parameter Beta Method from the true density function $f(\theta)$, the estimated item characteristic functions in both Degree 3 and 4 Cases are almost as good as the criterion item characteristic functions. This fact makes us wonder how strongly the discrepancies between the estimated probability density function of θ by the Conditional P.D.F. Method and

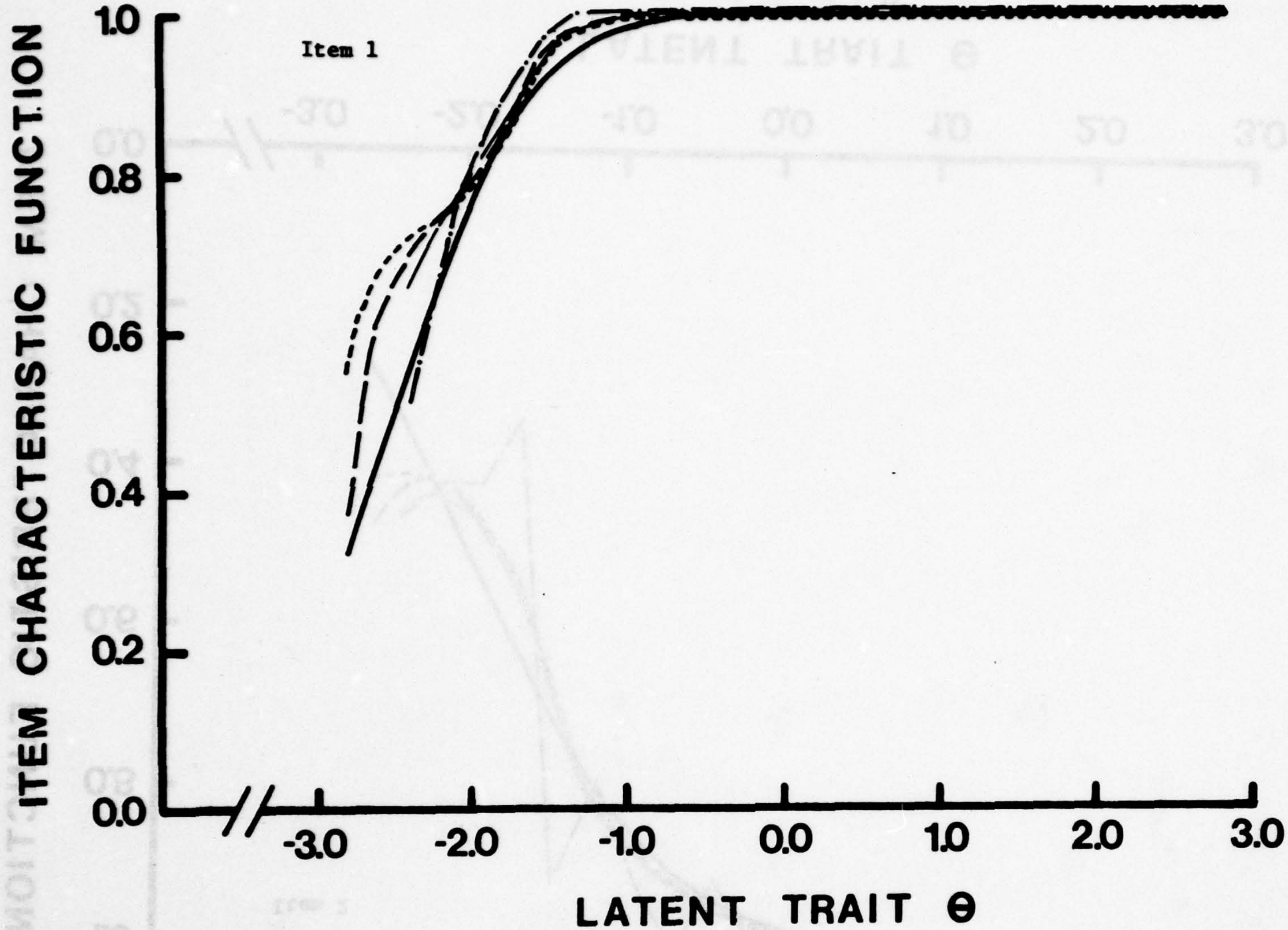

FIGURE 6-3

Estimated Item Characteristic Functions by the Conditional P.D.F. Method in Degree 3 Case (Broken Curve) and in Degree 4 Case (Dashed Curve), with the Criterion Item Characteristic Function (Thin Solid Curve), the Frequency Ratios of θ (Broken and Dotted Line) and the True Item Characteristic Function (Thick Solid Curve)

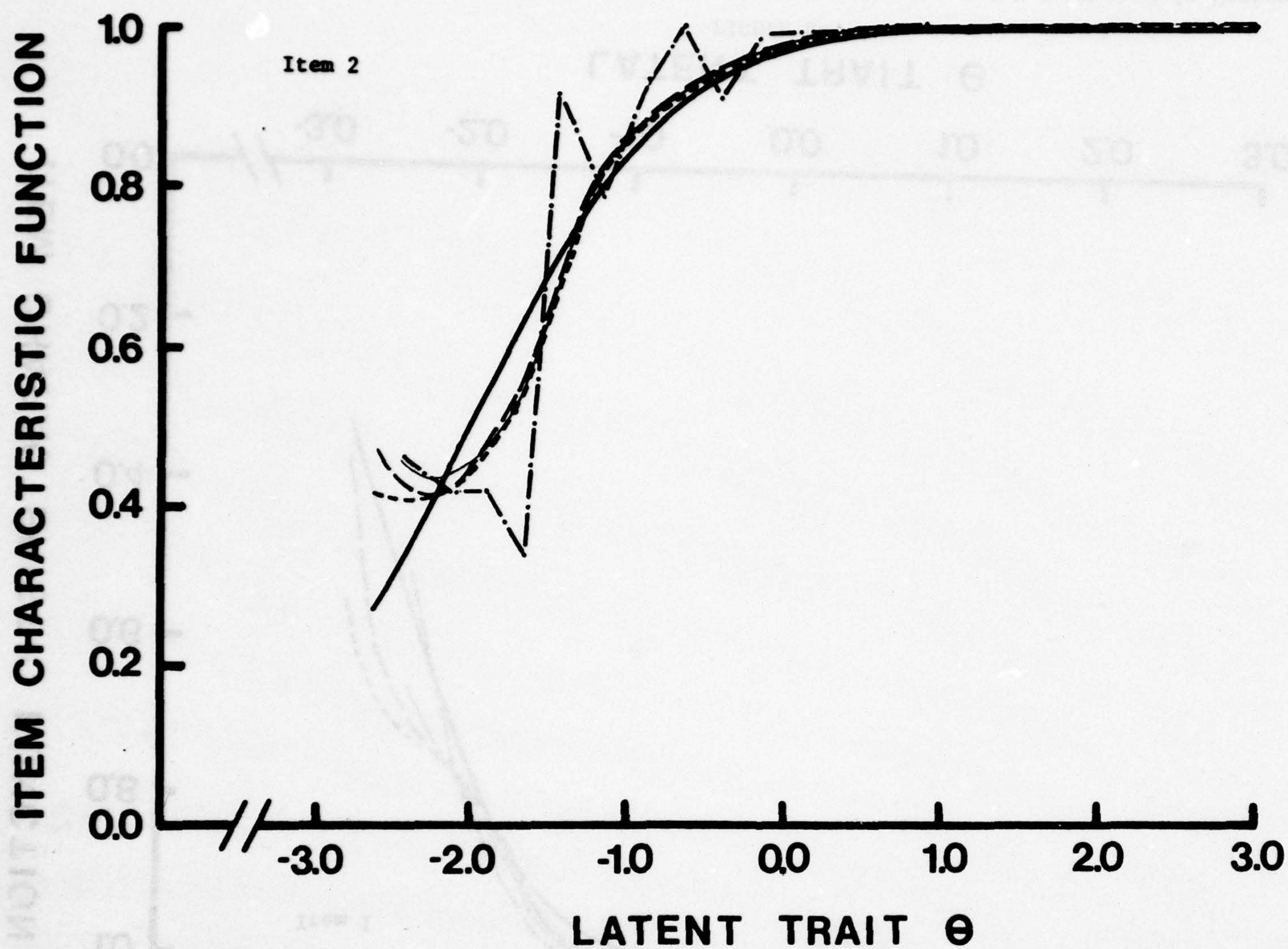

FIGURE 6-3 (Continued)

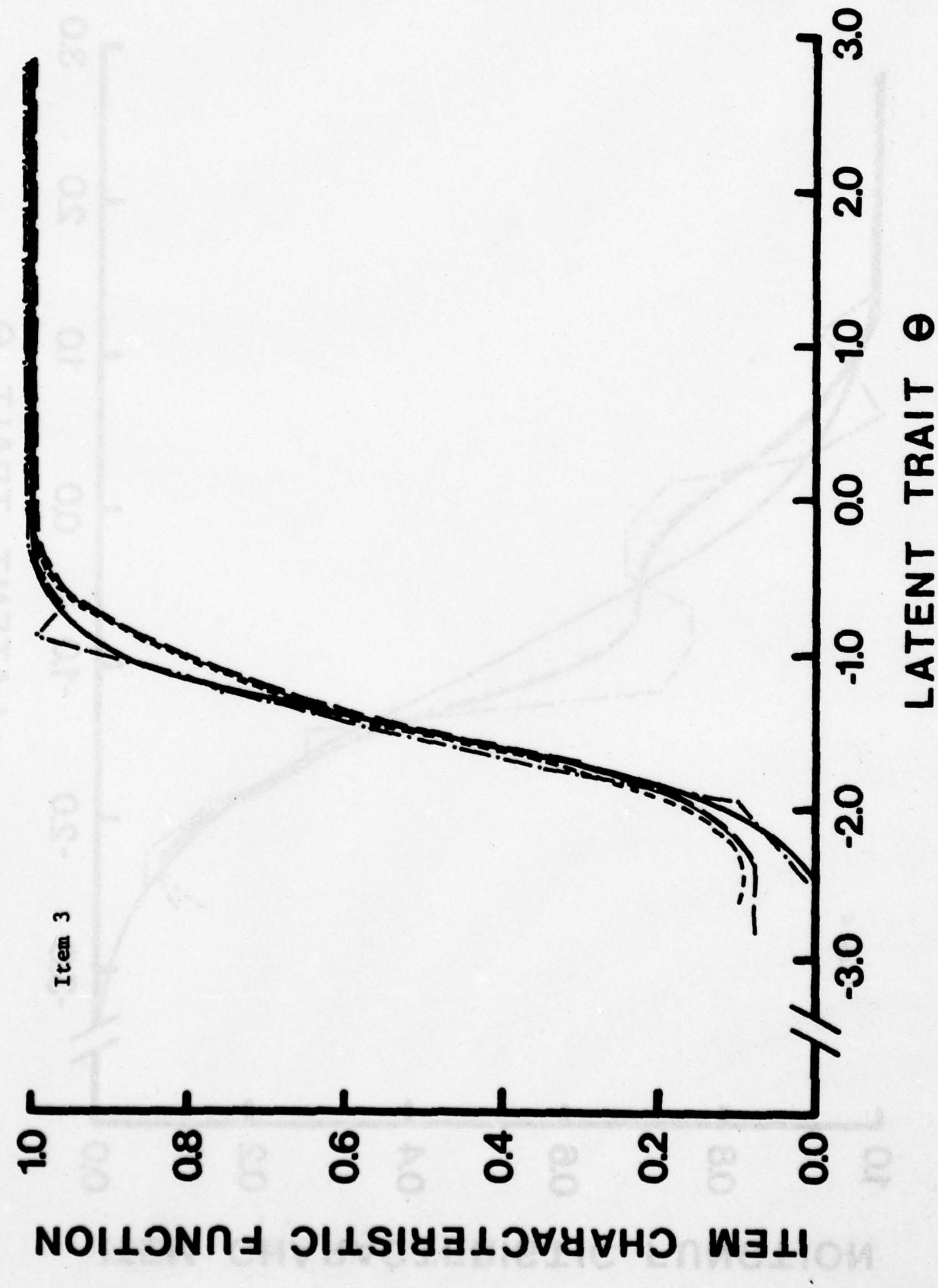

FIGURE 6-3 (Continued)

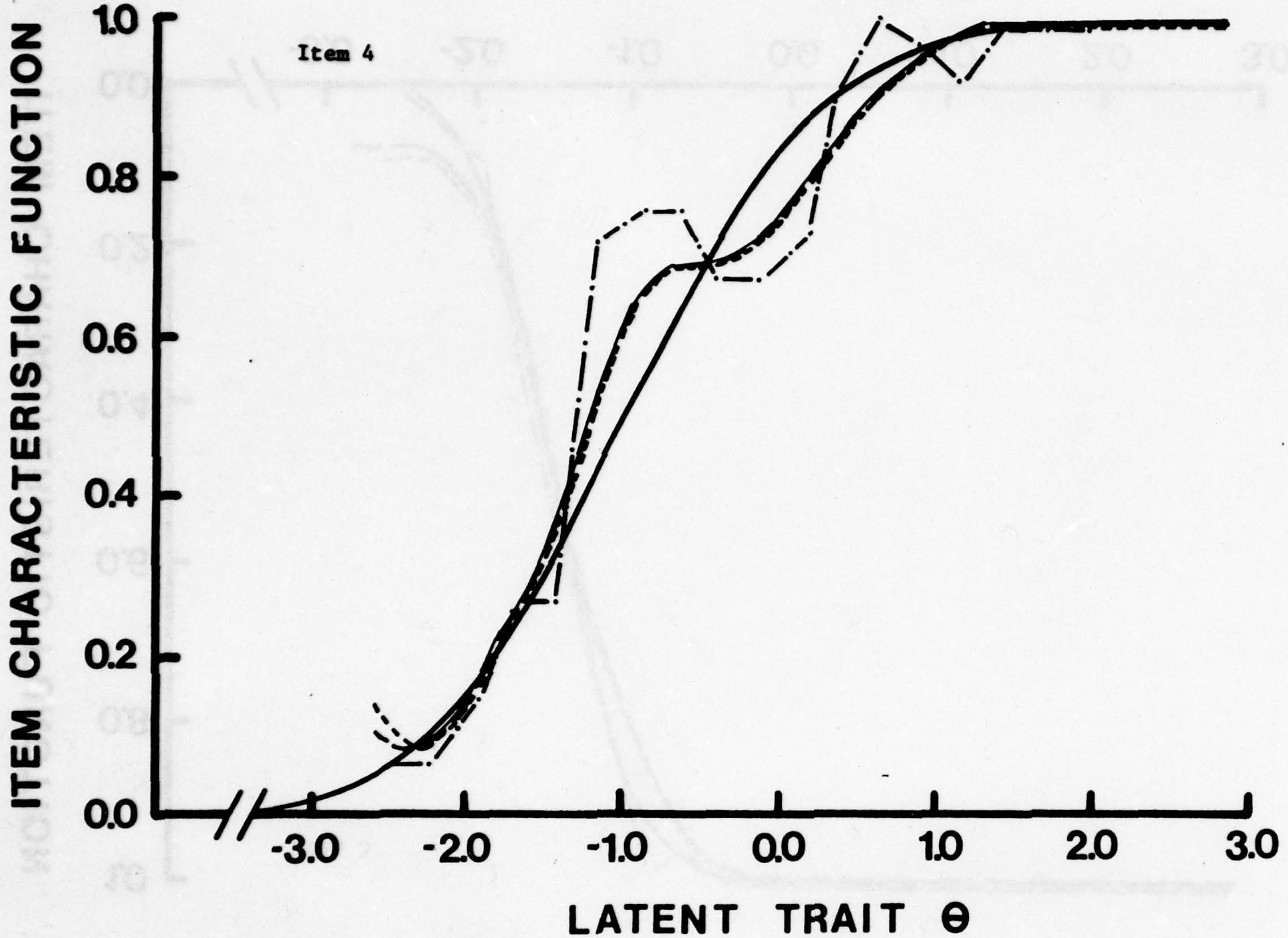

FIGURE 6-3 (Continued)

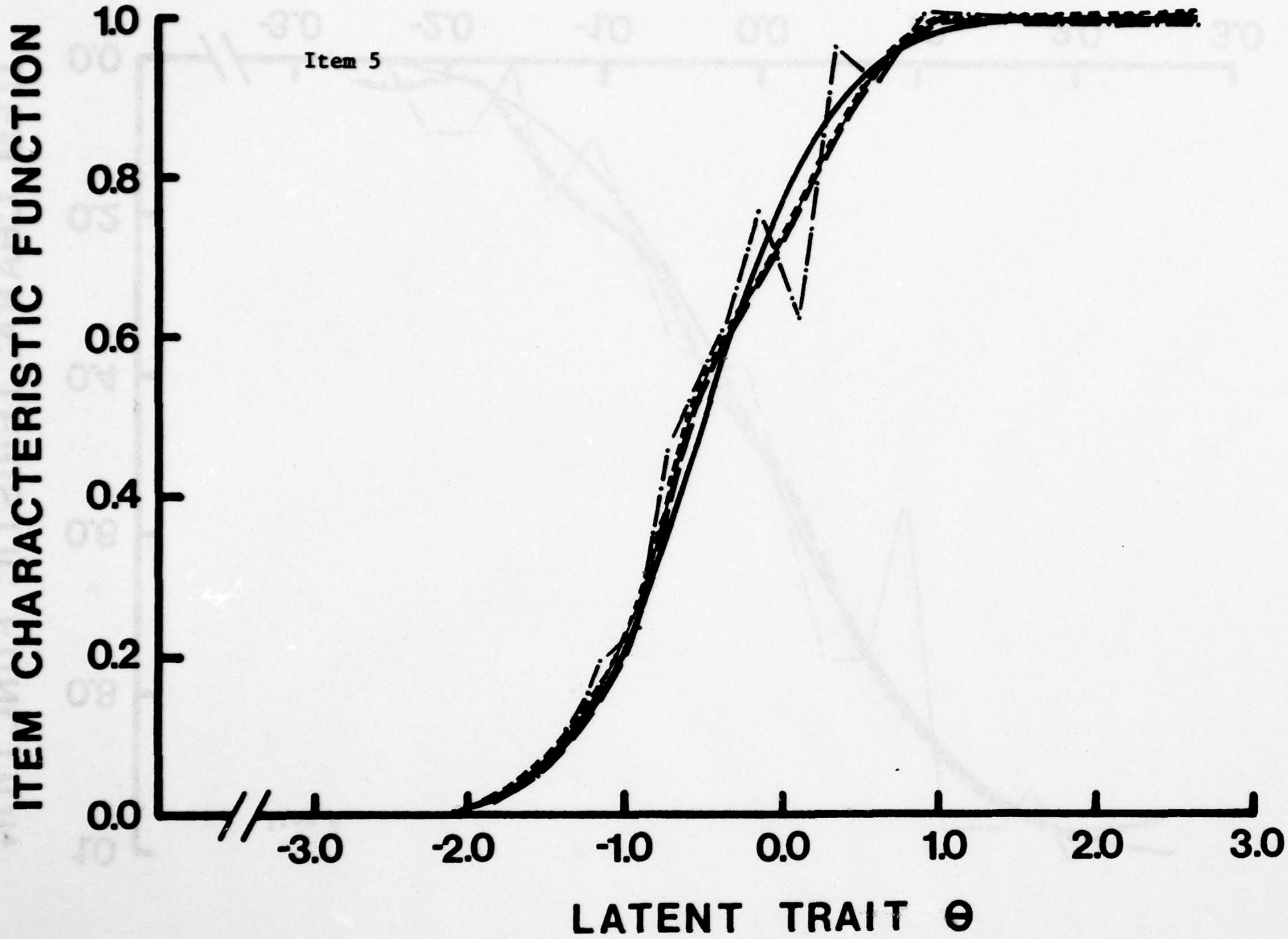

FIGURE 6-3 (Continued)

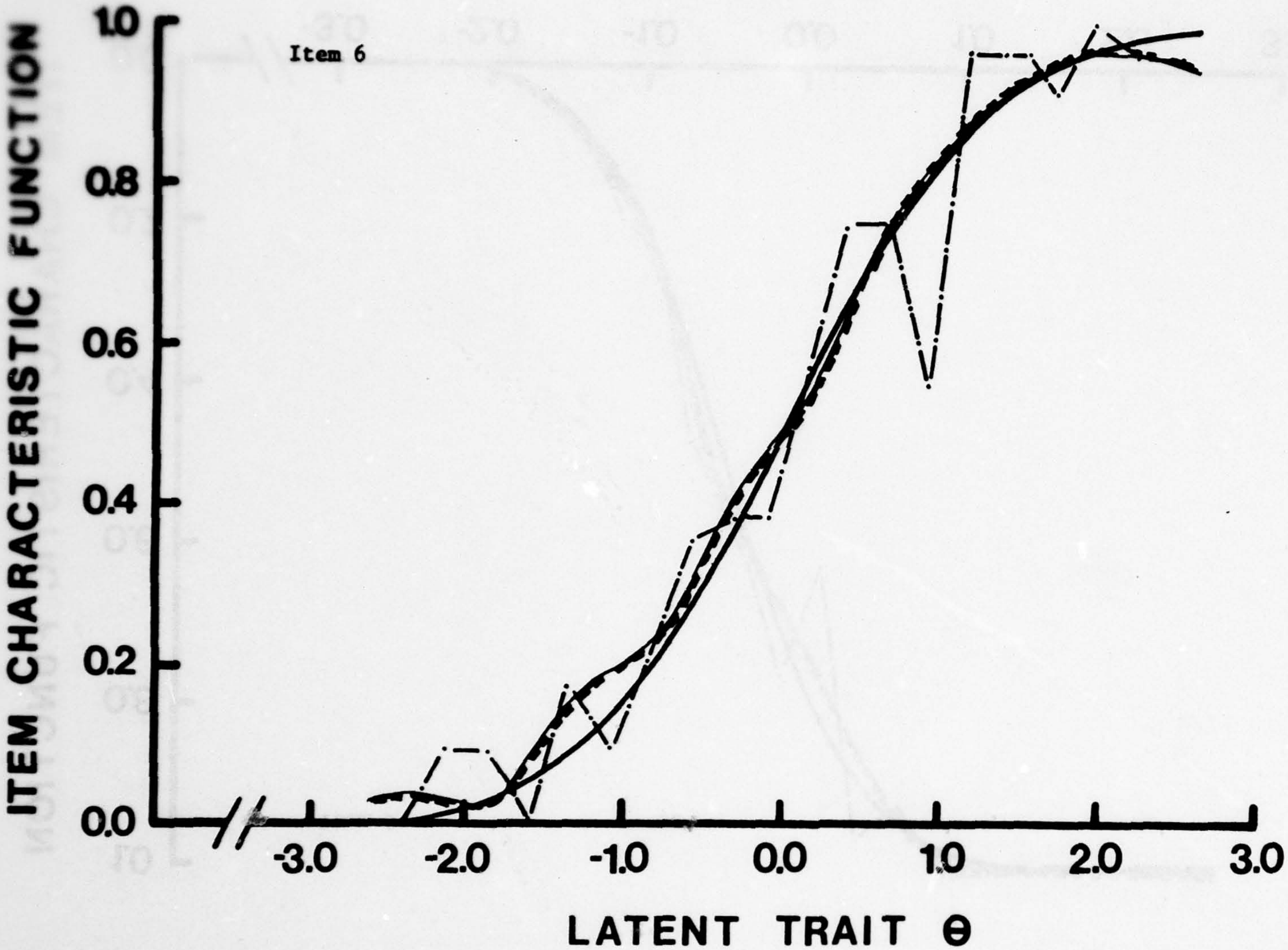

FIGURE 6-3 (Continued)

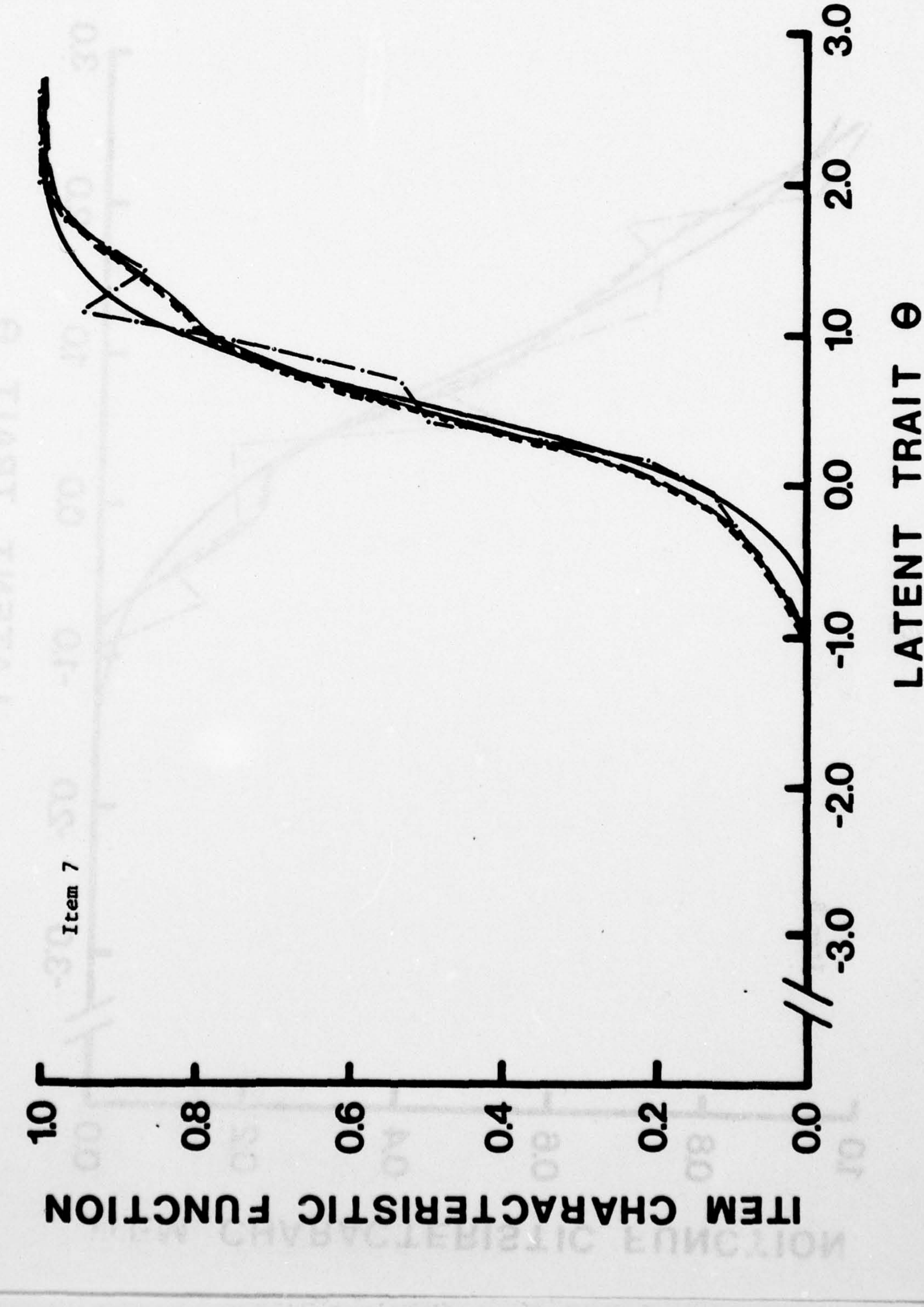

FIGURE 6-3 (Continued)

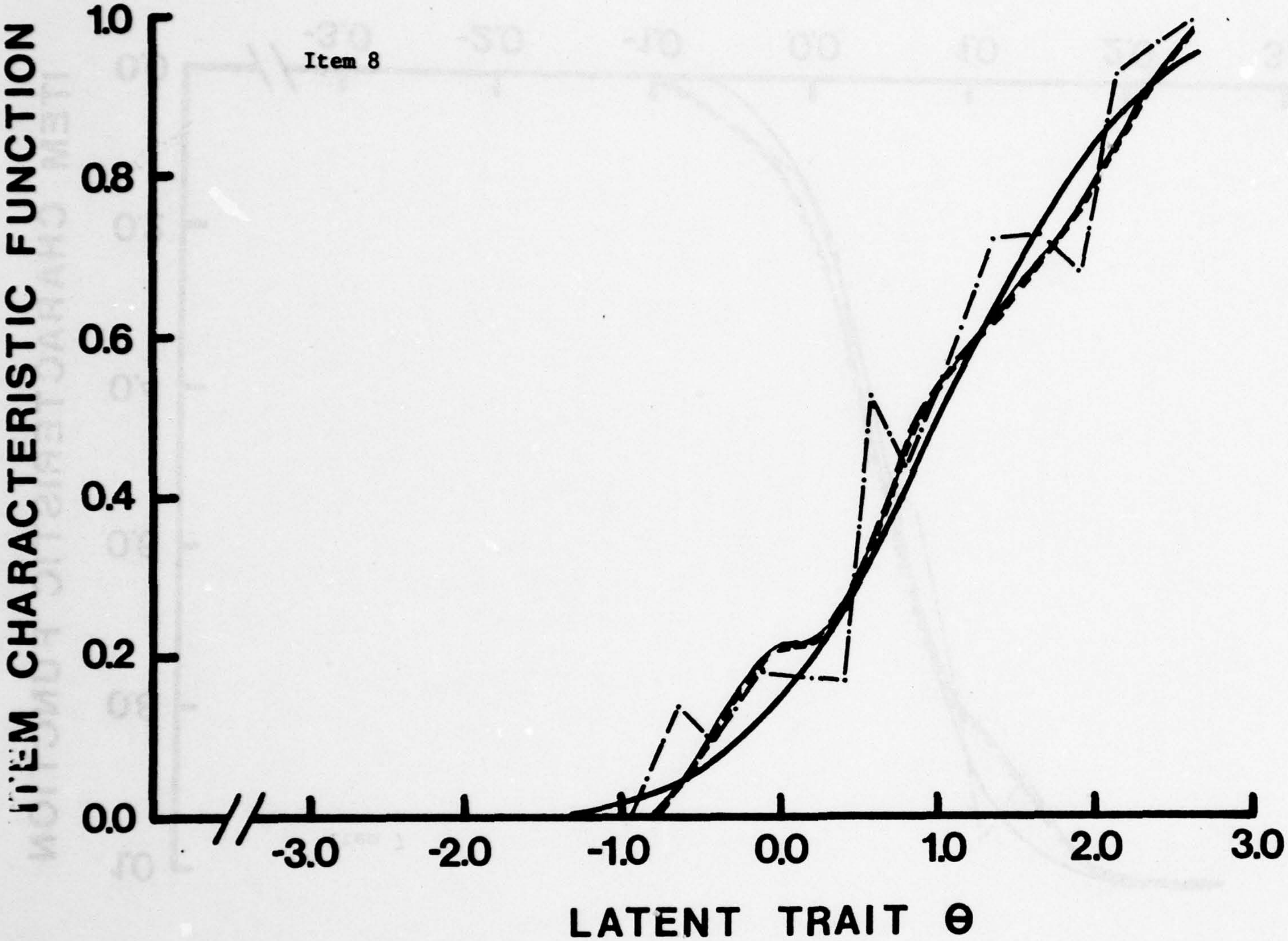

FIGURE 6-3 (Continued)

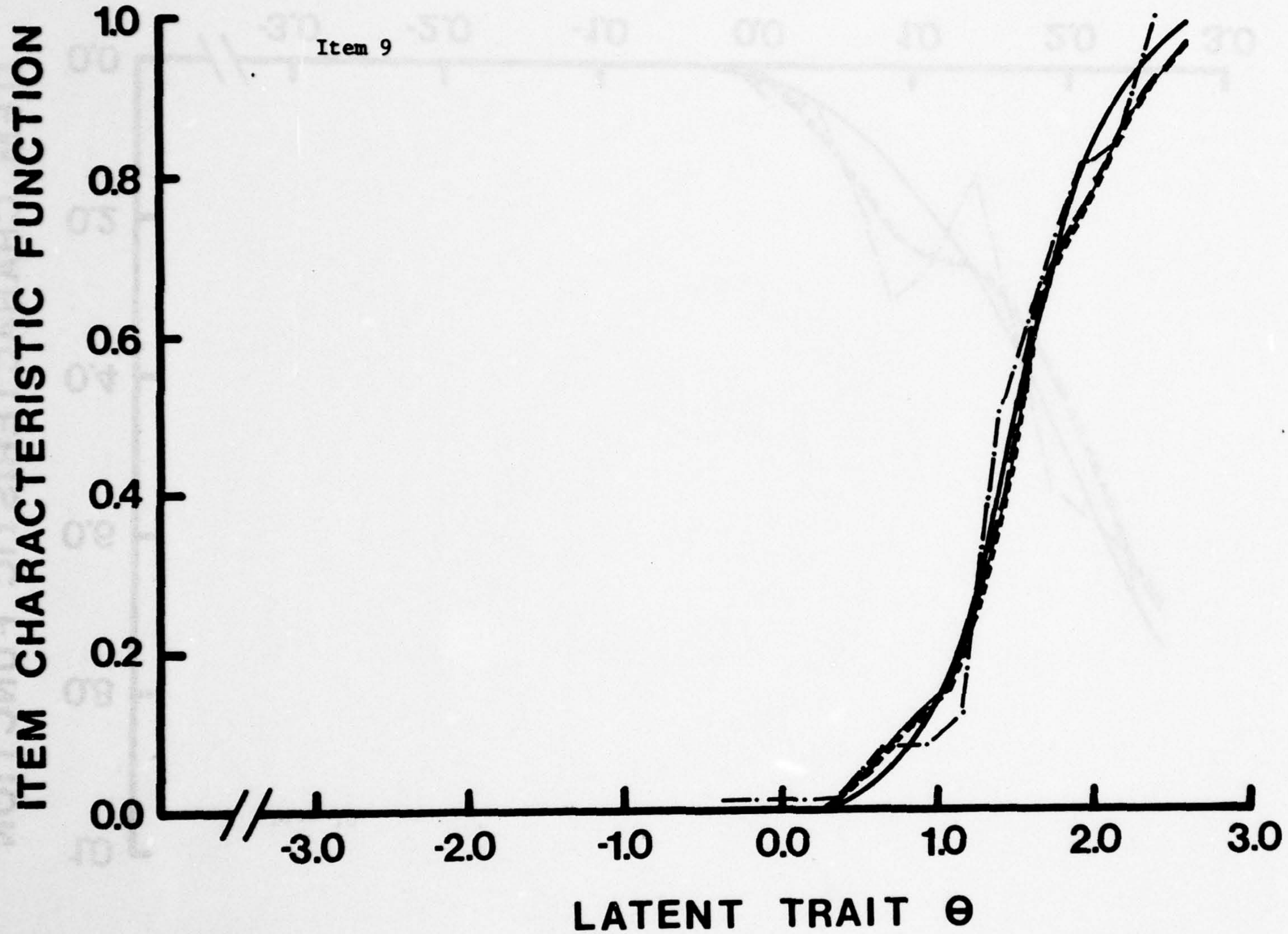

FIGURE 6-3 (Continued)

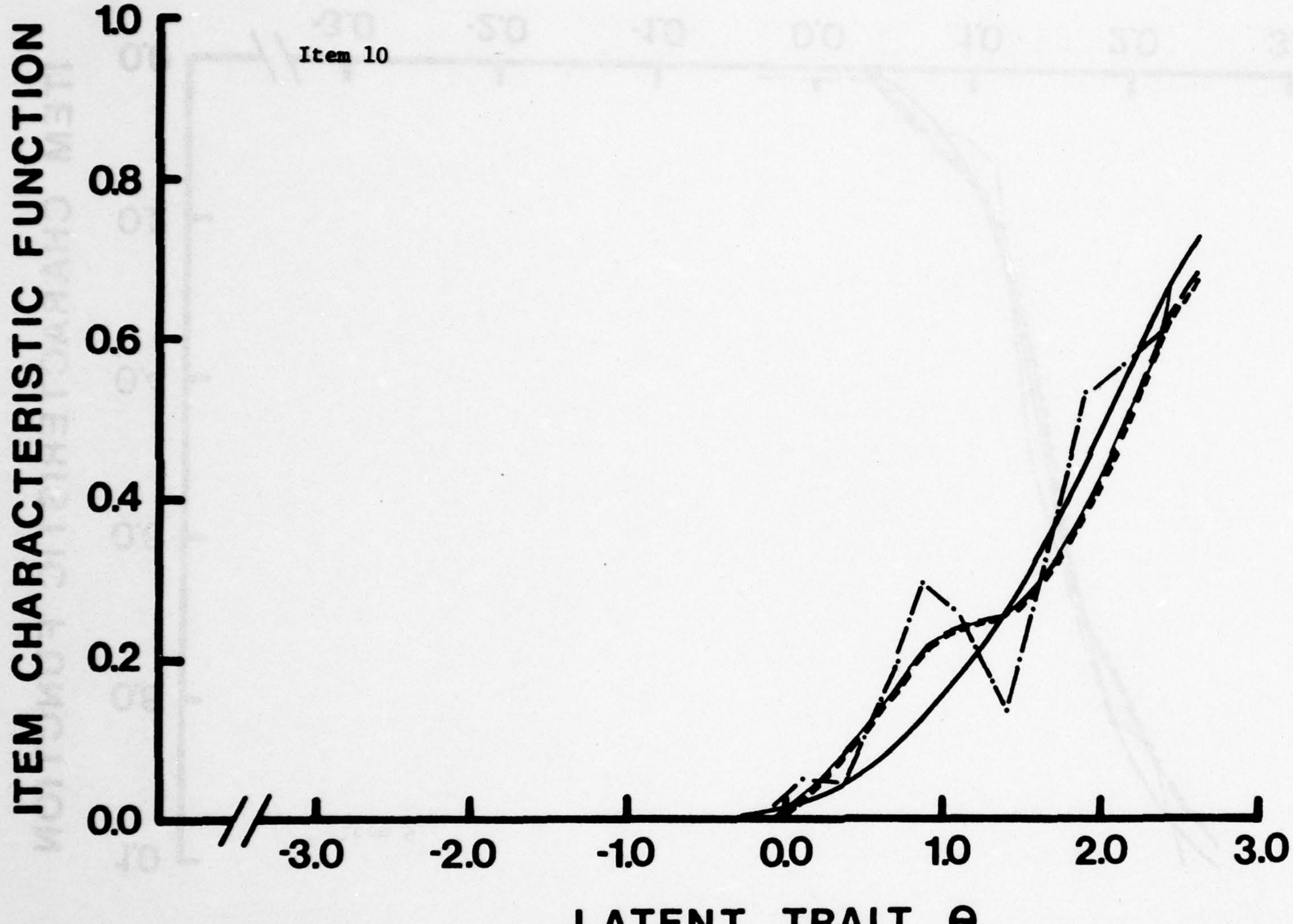

FIGURE 6-3 (Continued)

the theoretical probability density function, which are illustrated in Figures 6-1 and 6-2, are really affected by the way we approximated the conditional distribution by a Beta distribution; or are they more affected by the sampling fluctuations of the item responses? To answer this question, Figure 6-4 presents the function given by (4.2), where $\hat{\phi}(\theta|\hat{\theta}_s)$ is replaced by the true conditional density function obtained by (5.1). It is clear from this figure that, although at the extreme ends of θ this curve shows a substantial difference from the two curves obtained by the Conditional P.D.F. Method of the Two-Parameter Beta Method, for the intermediate values of θ it shows fairly similar variations as are seen in the results of the other two curves: evidence which confirms the above suspicion.

The usual simple least square method was applied in estimating the discrimination and difficulty parameters of the normal ogive model, a_g and b_g. Table 6-1 and Table 6-2 present the results in Degree 3 and 4 Cases and for the criterion item characteristic functions, using the interval of the ordinate 0.05 and 0.95 inclusive, within the interval of θ -4.0 and 4.0 inclusive. We notice that for some items, like item 2, the values of the estimated parameters are substantially different from the true parameters, and also from those for the criterion item characteristic functions, whereas for some other items, like item 8, they are close.

These large differences are due to the deviations of the estimated item characteristic functions at the extreme values of θ, as is hinted in Figure 6-3. Tables 6-3 and 6-4 present similar estimates

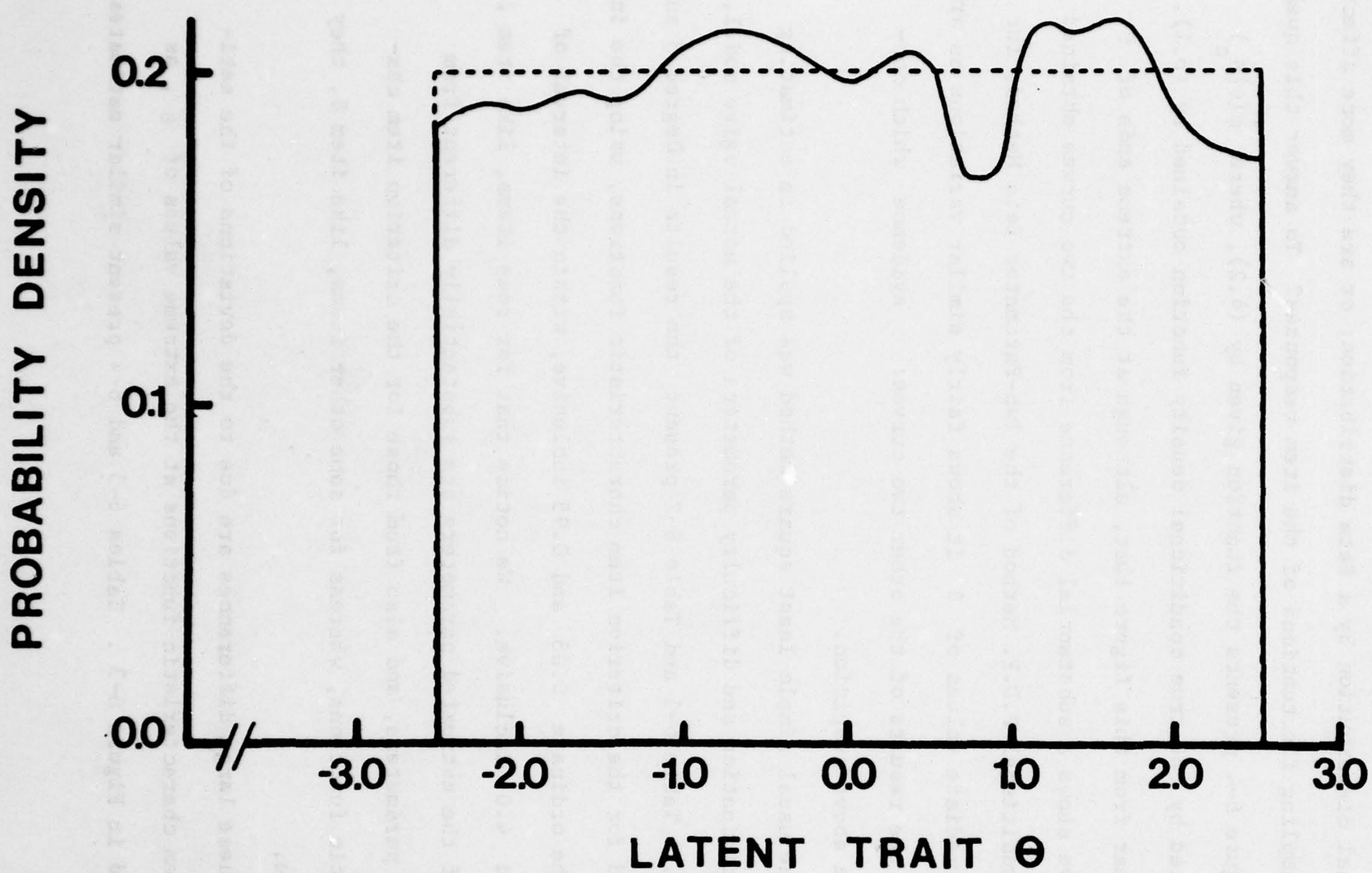

FIGURE 6-4

Approximated Marginal Density Function of θ Obtained from the 500 Conditional Density Functions of θ, Given Its Maximum Likelihood Estimate (Solid Curve) and the Theoretical Density Function of θ (Dashed Line)

obtained by using the interval of θ , -2.4 and 2.4 inclusive, instead of [-4.0, 4.0] . We note that the values of the estimated parameters for the criterion item characteristic functions are identical with those in Tables 6-1 and 6-2 . This is from the fact that the functions assume zero and unity outside the range of θ -2.5 through 2.5 respectively. The estimates of a_g and b_g in Degree 3 and 4 Cases shown in these two tables are, as a whole, much closer to the true parameter values, and quite comparable with those obtained from the criterion item characteristic functions. For additional information, the complete lists of the estimated parameters, which include the cases where the interval of the ordinate is 0.15 through 0.85 , 0.10 through 0.90 , and 0.01 through 0.99 , are presented in Appendix II, as Tables A-2-1 and A-2-2 when the range of θ is -4.0 through 4.0 , and as Tables A-2-3 and A-2-4 when the range of θ is -2.4 through 2.4 .

TABLE 6-1

Discrimination Parameter and Its Estimates of Each of the Ten Binary Items Obtained from the Estimated Item Characteristic Functions by the Conditional P.D.F. Method of the Two-Parameter Beta Method, with the Corresponding Estimate Obtained from the Criterion Item Characteristic Function: The Range of Each Estimated Item Characteristic Function Used Is [0.05, 0.95], within the Interval of θ, [-4.0, 4.0]

METHOD / ITEM	TRUE a_g	$\hat{a}_g$ from DGR.3 0.05-0.95	$\hat{a}_g$ from DGR.4 0.05-0.95	$\hat{a}_g$ from CRITERION 0.05-0.95
1	1.5	1.323	1.289	1.400_5
2	1.0	0.349	0.608	1.024
3	2.5	1.314	1.222	1.788
4	1.0	0.787	0.812	0.868
5	1.5	1.394	1.378	1.368
6	1.0	0.906	0.602	0.895
7	2.0	1.489	1.480	1.473
8	1.0	0.911	0.919	0.886
9	2.0	1.676	1.709	1.716
10	1.0	0.659	0.724	0.725

The number of intervals used in estimation is shown as a subscript when it is less than 6.

TABLE 6-2

Difficulty Parameter and Its Estimates of Each of the Ten Binary Items Obtained from the Estimated Item Characteristic Functions by the Conditional P.D.F. Method of the Two-Parameter Beta Method, with the Corresponding Estimate Obtained from the Criterion Item Characteristic Function: The Range of Each Estimated Item Characteristic Function Used Is [0.05, 0.95], within the Interval of θ, [-4.0, 4.0].

ITEM \ METHOD	TRUE b_g	$\hat{b}_g$ from DGR.3 0.05-0.95	$\hat{b}_g$ from DGR.4 0.05-0.95	$\hat{b}_g$ from CRITERION 0.05-0.95
1	-2.5	-2.684	-2.737	-2.651_5
2	-2.0	-3.310	-2.478	-2.002
3	-1.5	-1.527	-1.552	-1.507
4	-1.0	-1.071	-1.056	-1.005
5	-0.5	-0.462	-0.466	-0.472
6	0.0	-0.074	0.047	-0.075
7	0.5	0.523	0.528	0.527
8	1.0	0.958	0.953	0.981
9	1.5	1.505	1.506	1.502
10	2.0	2.259	2.170	2.118

The number of intervals used in estimation is shown as a subscript when it is less than 6.

TABLE 6-3

Discrimination Parameter and Its Estimates of Each of the Ten Binary Items Obtained from the Estimated Item Characteristic Functions by the Conditional P.D.F. Method of the Two-Parameter Beta Method, with the Corresponding Estimate Obtained from the Criterion Item Characteristic Function: The Range of Each Estimated Item Characteristic Function Used Is [0.05, 0.95], within the Interval of θ , [-2.4, 2.4].

ITEM \ METHOD	TRUE a_g	$\hat{a}_g$ from DGR.3 0.05-0.95	$\hat{a}_g$ from DGR.4 0.05-0.95	$\hat{a}_g$ from CRITERION 0.05-0.95
1	1.5	1.119_5	1.112_5	1.400_5
2	1.0	1.051	1.067	1.024
3	2.5	1.785	1.777	1.788
4	1.0	0.887	0.875	0.868
5	1.5	1.394	1.378	1.368
6	1.0	0.906	0.897	0.895
7	2.0	1.489	1.480	1.473
8	1.0	0.911	0.919	0.886
9	2.0	1.676	1.709	1.716
10	1.0	0.762	0.769	0.725

The number of intervals used in estimation is shown as a subscript when it is less than 6.

TABLE 6-4

Difficulty Parameter and Its Estimates of Each of the Ten Binary Items Obtained from the Estimated Item Characteristic Functions by the Conditional P.D.F. Method of the Two-Parameter Beta Method, with the Corresponding Estimate Obtained from the Criterion Item Characteristic Function: The Range of Each Estimated Item Characteristic Function Used Is [0.05, 0.95], within the Interval of θ, [-2.4, 2.4].

ITEM \ METHOD	TRUE b_g	$\hat{b}_g$ from DGR.3 0.05-0.95	$\hat{b}_g$ from DGR.4 0.05-0.95	$\hat{b}_g$ from CRITERION 0.05-0.95
1	-2.5	-2.815_5	-2.837_5	-2.651_5
2	-2.0	-1.968	-1.965	-2.002
3	-1.5	-1.494	-1.505	-1.507
4	-1.0	-0.992	-1.001	-1.005
5	-0.5	-0.462	-0.466	-0.472
6	0.0	-0.074	-0.075	-0.075
7	0.5	0.523	0.528	0.527
8	1.0	0.958	0.953	0.981
9	1.5	1.505	1.506	1.502
10	2.0	2.108	2.104	2.118

The number of intervals used in estimation is shown as a subscript when it is less than 6.

VII Discussion and Conclusion

Two variations of the Two-Parameter Beta Method for estimating the operating characteristics of the item score categories have been presented, and used for the same simulated data of five hundred hypothetical examinees, whose ability distributes uniformly, in estimating the item characteristic functions of the ten binary items which follow the normal ogive model. The criterion operating characteristic has been introduced, and used for evaluating the results of the Conditional P.D.F. Method.

In spite of the fact that the approximation by a polynomial of degree 3 has a theoretical disadvantage compared with the approximation by a polynomial of degree 4, so far we have found very little evidence to support Degree 4 Case in preference to Degree 3 Case. Any hasty conclusions should be avoided, however, until we have tried these methods on different types of data, e.g., those of a smaller or larger number of examinees, those on different ability distributions, those for estimating the operating characteristics other than those in the normal ogive model of the dichotomous response level, and so forth.

The results of the Conditional P.D.F. Method suggest that the a priori set functional formula for the conditional distribution of ability, given its maximum likelihood estimate, may not be so important as it looks, and some other functional formulas might serve as well. Investigations in this direction will be made in the near future.

The fact that in both Degree 3 and 4 Cases the estimated item characteristic function turned out to be extremely close to the criterion item characteristic function indicates the limitation of

the Conditional P.D.F. Method, as well as its accuracy of estimating the operating characteristics. For this reason, not only we should try to refine the present method, but also we should explore new methods which will eliminate the strong effect of the sampling fluctuations of the item responses. This can be done by approximating the bivariate distribution of ability and its maximum likelihood estimate for each item score group, the idea which underlies the Normal Approximation Method. Investigations will be made in this direction also in the near future.

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ESTIMATION OF THE OPERATING CHARACTERISTICS OF ITEM RESPONSE CA--ETC(U)
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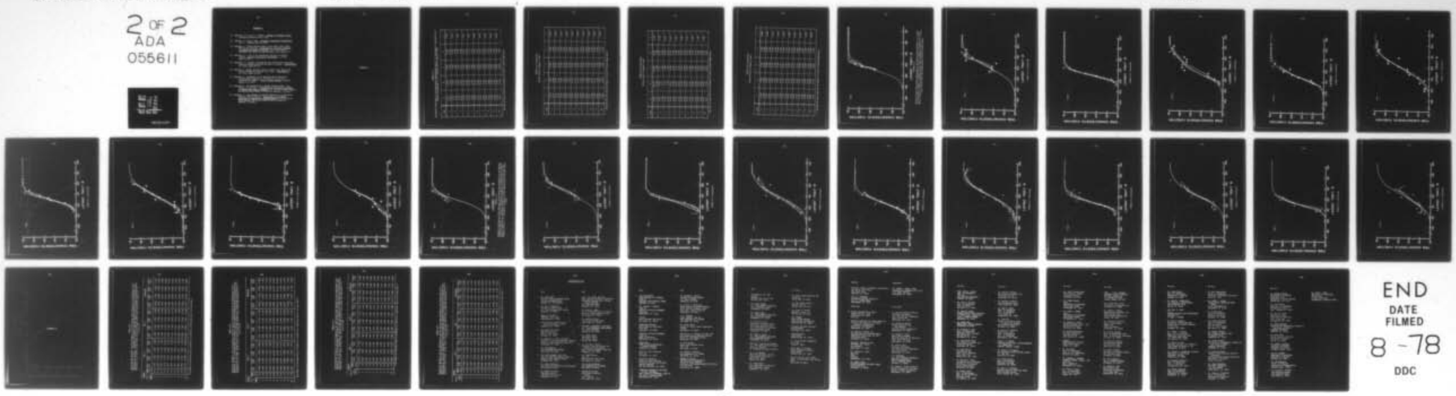

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APPENDIX I

TABLE A-1-1

Estimated Coefficients of the Polynomial Graduating the Set of $\tilde{\theta}$, Converted to Approximate the Relative Frequencies of $\tilde{\theta}$: Failure Group in Degree 3 Case

Item	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{\nu}$
1	17.973	39.936	31.547	10.340	1.202
	-0.572	0.407	1.135	0.304	
2	0.131	0.692	1.186	0.383	0.016
	0.113	0.575	1.007	0.288	
3	1.708	5.676	6.297	2.496	0.322
	-0.243	-0.129	0.537	0.186	
4	0.132	-0.132	0.055	-0.017	-0.019
	0.114	-0.203	0.085	0.054	
5	0.119	-0.279	0.107	0.116	0.015
	0.138	-0.242	0.056	0.052	
6	0.205	-0.154	0.001	0.018	-0.002
	0.217	-0.162	-0.014	0.021	
7	0.252	-0.163	-0.040	0.024	0.004
	0.245	-0.144	-0.027	0.017	
8	0.237	-0.091	-0.018	0.010	-0.001
	0.242	-0.095	-0.024	0.011	
9	0.256	-0.048	-0.028	0.003	-0.001
	0.260	-0.051	-0.032	0.004	
10	0.231	-0.037	-0.015	0.004	-0.001
	0.241	-0.035	-0.025	0.004	

Two rows for each item are for Degree 3-4 and 3-3 Cases respectively.

TABLE A-1-1 (Continued)

Success Group in Degree 3 Case

Item	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{\nu}$
1	0.219	0.004	-0.005	0.000	-0.002
	0.237	0.006	-0.024	0.000	
2	0.238	0.034	-0.021	-0.003	-0.001
	0.242	0.035	-0.025	-0.004	
3	0.260	0.046	-0.032	-0.003	0.000
	0.260	0.047	-0.032	-0.003	
4	0.239	0.073	-0.018	-0.007	-0.001
	0.246	0.076	-0.026	-0.008	
5	0.250	0.152	-0.037	-0.021	0.003
	0.242	0.137	-0.025	-0.016	
6	0.212	0.144	-0.005	-0.017	-0.002
	0.220	0.151	-0.015	-0.019	
7	0.099	0.299	0.158	-0.163	0.025
	0.137	0.250	0.043	-0.047	
8	0.115	0.222	0.109	-0.083	0.007
	0.125	0.212	0.076	-0.052	
9	1.326	-4.757	5.594	-2.285	0.300
	-0.302	0.467	0.232	-0.116	
10	0.056	0.223	-0.048	0.114	-0.042
	0.087	-0.049	0.396	-0.129	

Two rows for each item are Degree 3-4 and 3-3 Cases respectively.

TABLE A-1-1 (Continued)

Failure Group in Degree 4 Case

Item	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{\nu}$
1	69.253	140.745	103.881	32.780	3.746
	7.772	14.669	8.996	1.692	
2	0.037	-0.065	-0.211	-0.477	-0.150
	0.130	0.744	1.184	0.333	
3	0.148	0.610	0.698	-0.030	-0.074
	0.357	1.426	1.687	0.436	
4	0.141	-0.037	0.046	-0.117	-0.051
	0.108	-0.200	0.104	0.060	
5	0.125	-0.222	0.110	0.064	-0.003
	0.122	-0.226	0.121	0.076	
6	0.199	-0.161	0.014	0.019	-0.005
	0.219	-0.168	-0.015	0.022	
7	0.246	-0.161	-0.029	0.023	0.002
	0.244	-0.155	-0.024	0.020	
8	0.230	-0.090	-0.005	0.010	-0.003
	0.243	-0.098	-0.023	0.012	
9	0.250	-0.042	-0.016	0.001	-0.003
	0.261	-0.050	-0.033	0.003	
10	0.221	-0.032	0.003	0.002	-0.004
	0.241	-0.036	-0.024	0.004	

Two rows for each item are for Degree 4-4 and 4-3 Cases respectively.

TABLE A-1-1 (Continued)

Success Group in Degree 4 Case

Item	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{\nu}$
1	0.209	0.006	0.012	0.000	-0.005
	0.235	0.008	-0.021	-0.001	
2	0.232	0.037	-0.008	-0.004	-0.003
	0.247	0.036	-0.026	-0.004	
3	0.256	0.051	-0.023	-0.004	-0.001
	0.263	0.051	-0.032	-0.004	
4	0.231	0.080	-0.002	-0.009	-0.004
	0.250	0.081	-0.027	-0.009	
5	0.239	0.150	-0.012	-0.018	-0.002
	0.246	0.157	-0.024	-0.021	
6	0.195	0.157	0.026	-0.019	-0.007
	0.227	0.165	-0.017	-0.022	
7	0.105	0.250	0.141	-0.085	-0.001
	0.104	0.253	0.144	-0.090	
8	0.127	0.142	0.080	0.033	-0.031
	0.100	0.172	0.201	-0.097	
9	0.956	-3.531	4.168	-1.580	0.178
	0.361	-1.401	1.705	-0.451	
10	0.005	0.459	-0.358	0.303	-0.083
	0.101	-0.122	0.520	-0.170	

Two rows for each item are for Degree 4-4 and 4-3 Cases respectively.

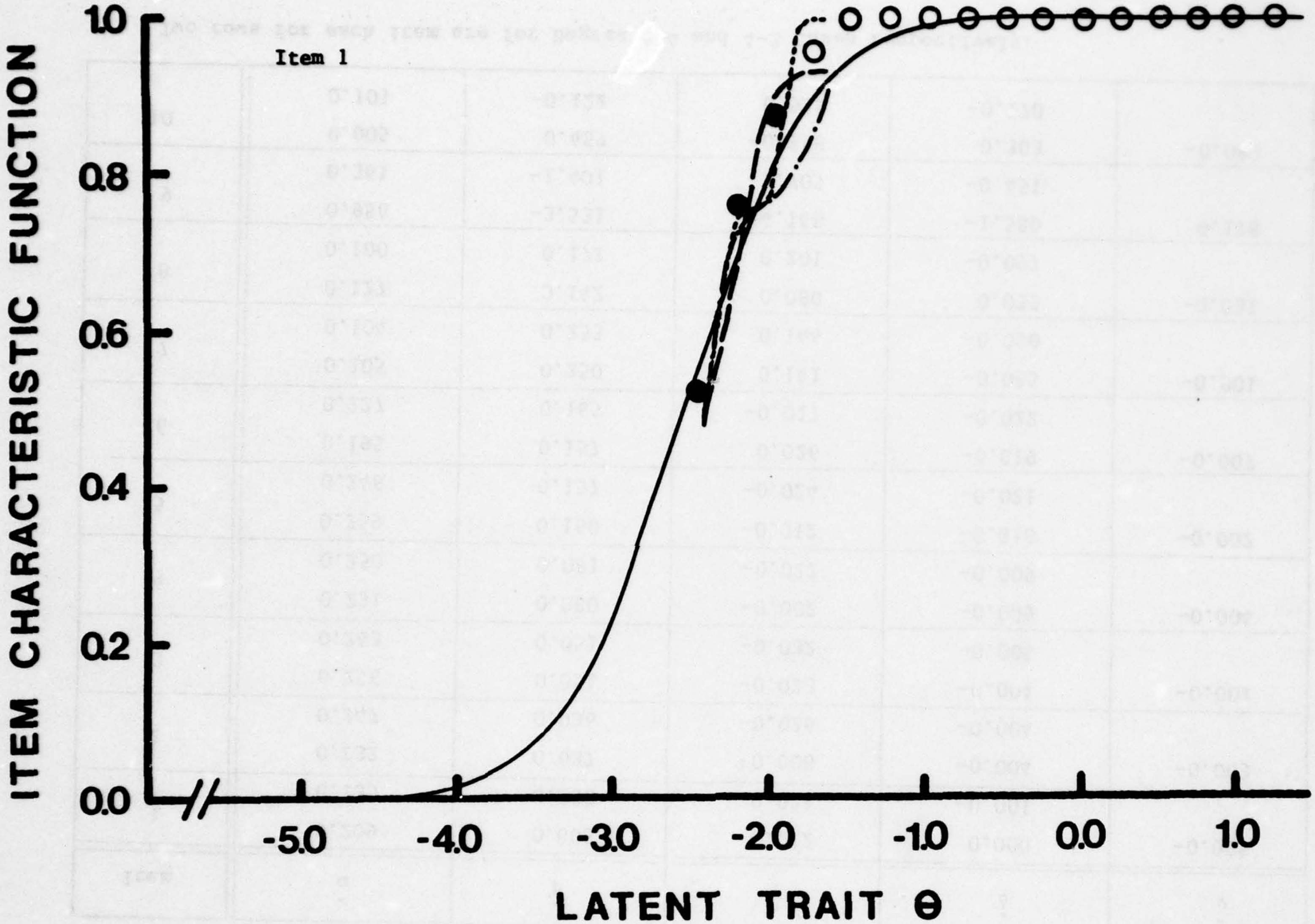
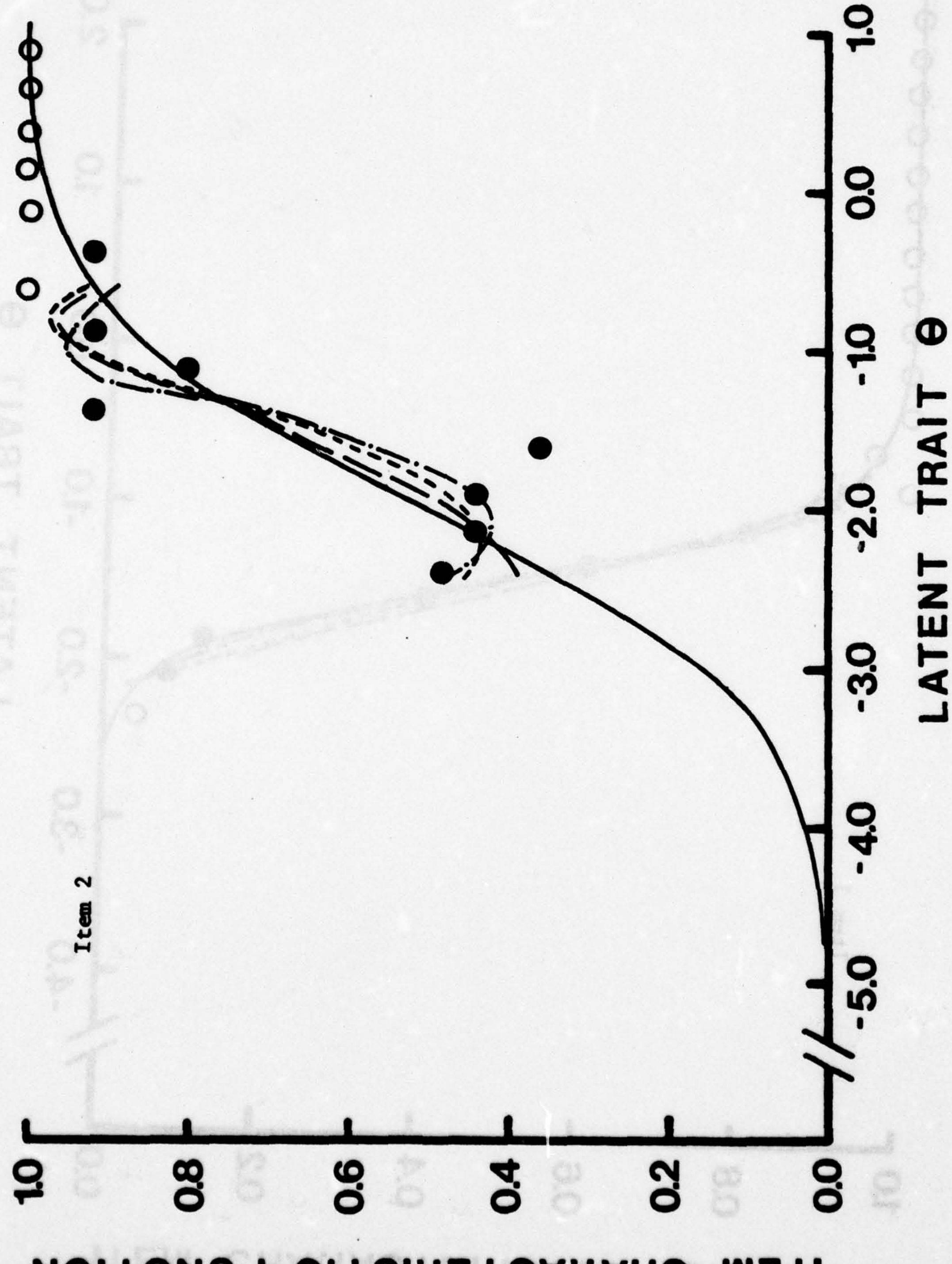


FIGURE A-1-1

Estimated Item Characteristic Functions by the Curve Fitting Method for θ, Using the Polynomials of Degree 3 (Broken Curve), Those of Degree 4 (Dashed Curve) and Those of Degree 5 (Broken and Dotted Curve), with the Frequency Ratios of θ (Circles) and the True Item Characteristic Function (Solid Curve)



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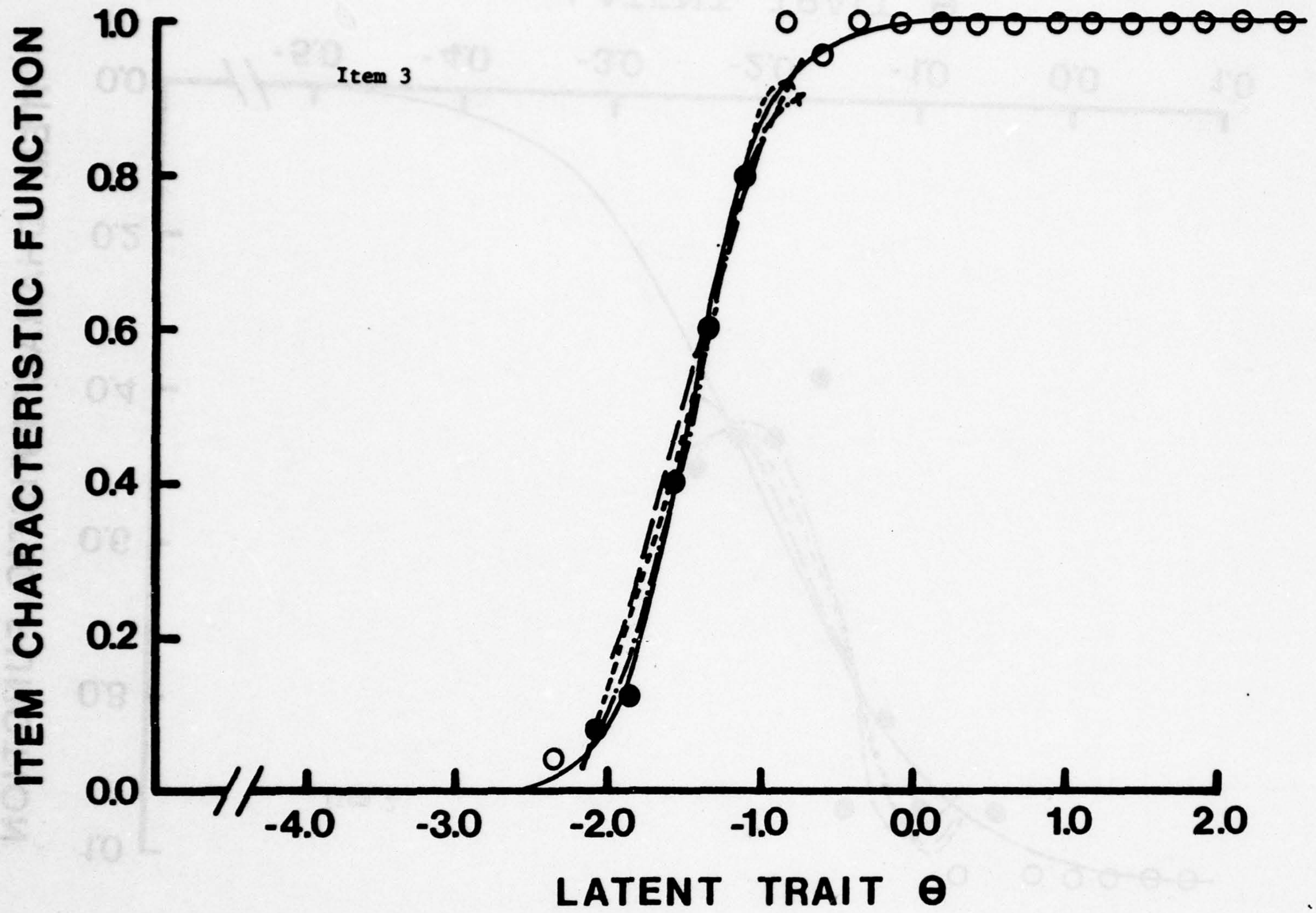

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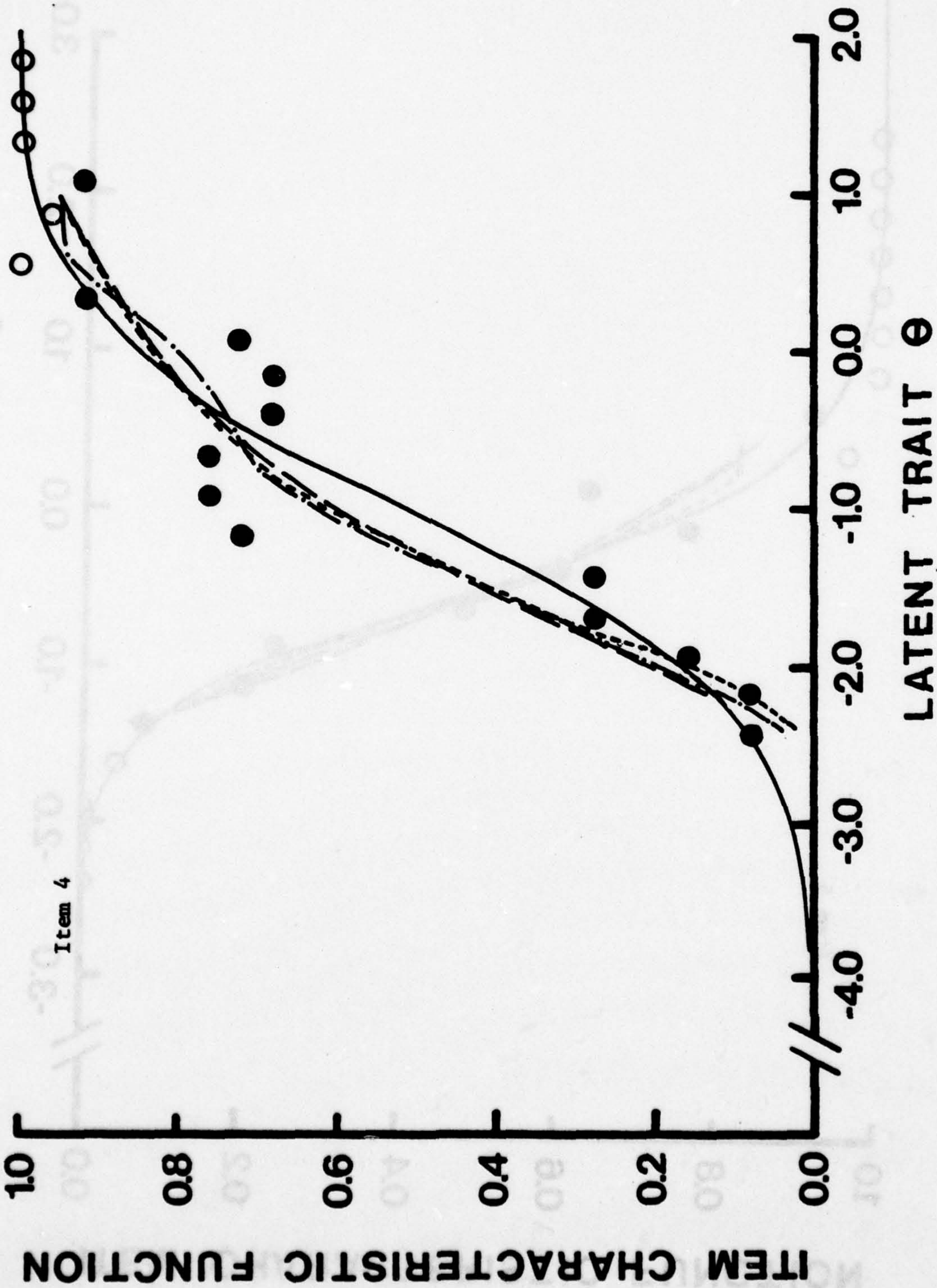

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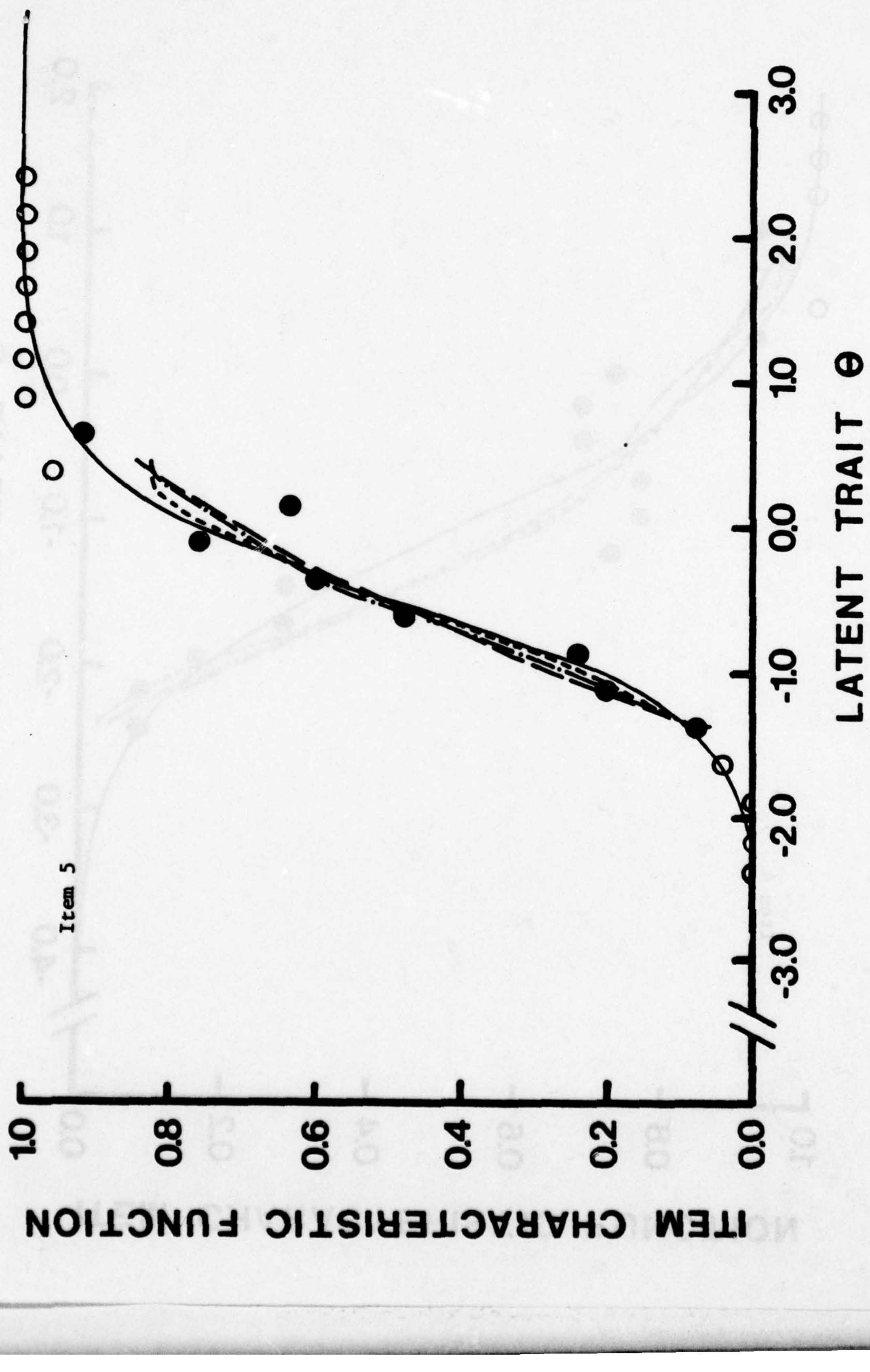

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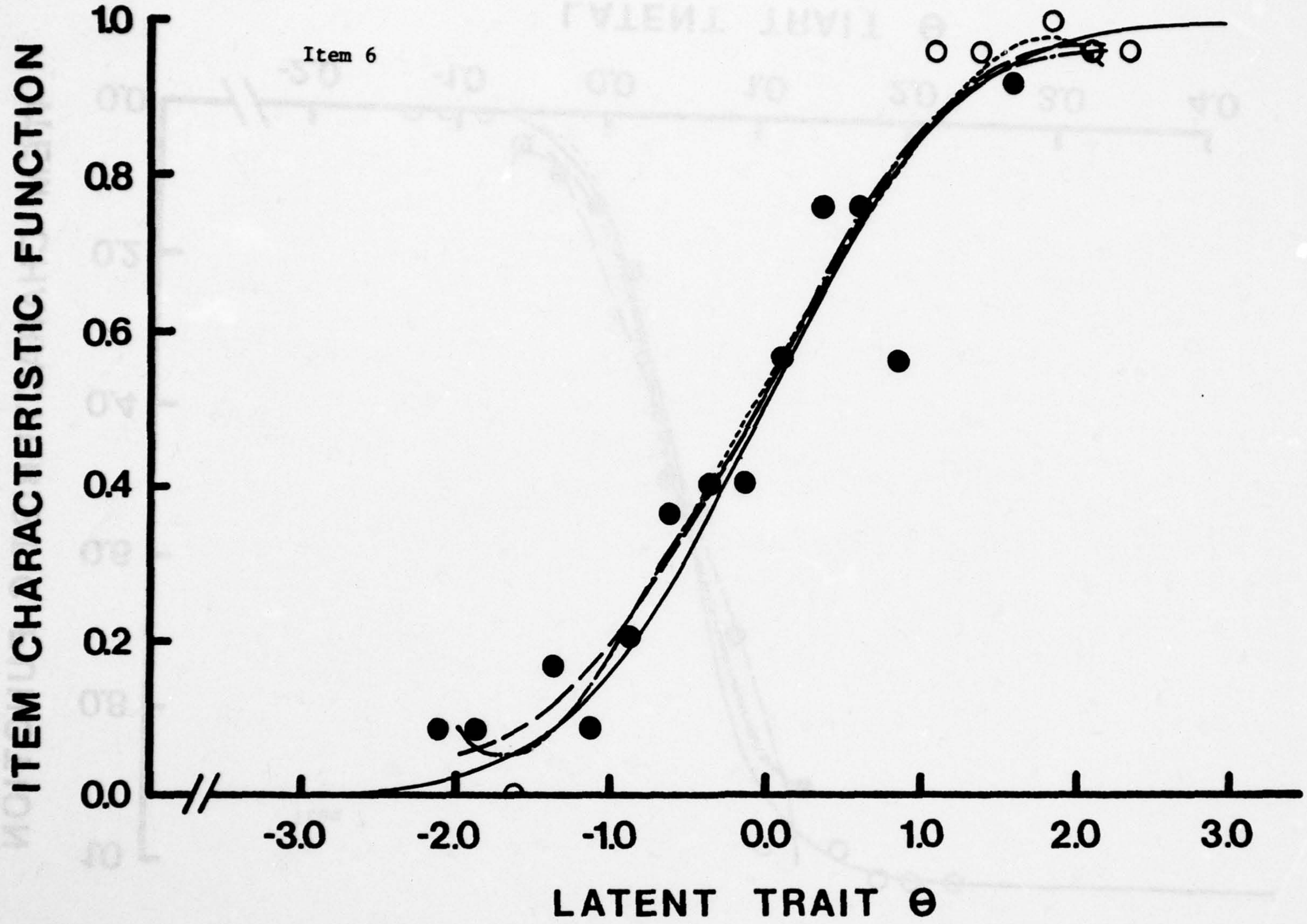

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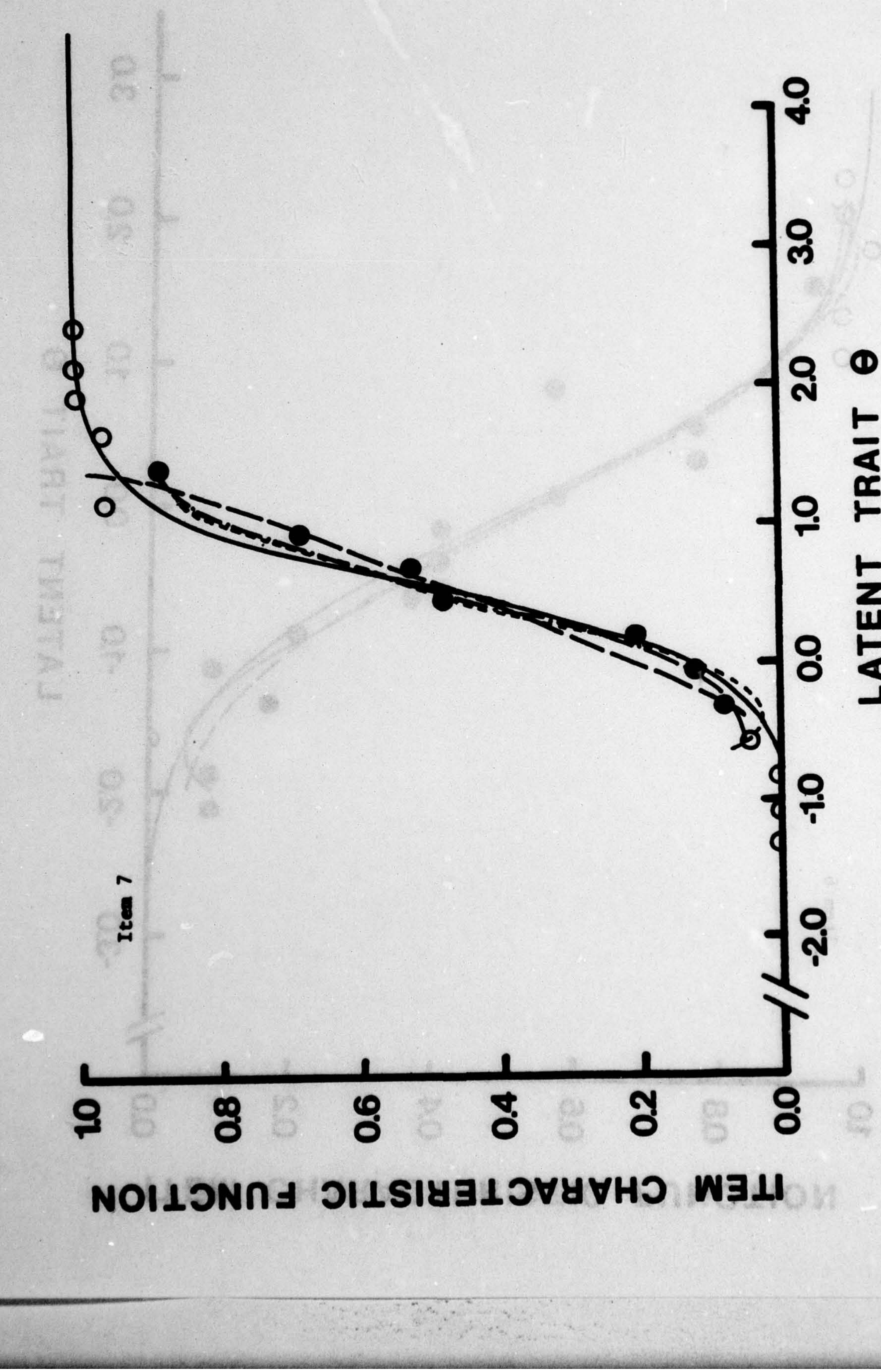

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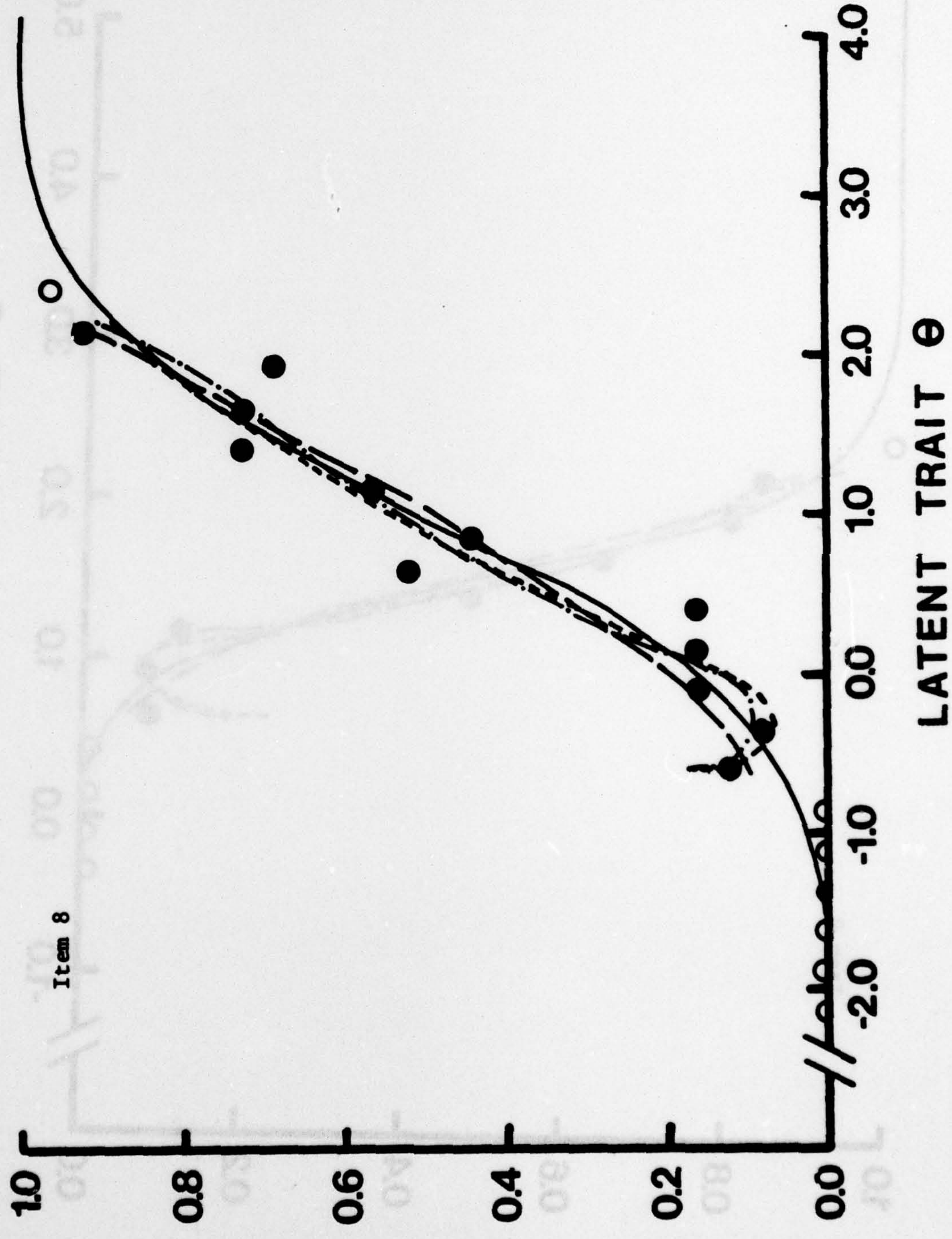

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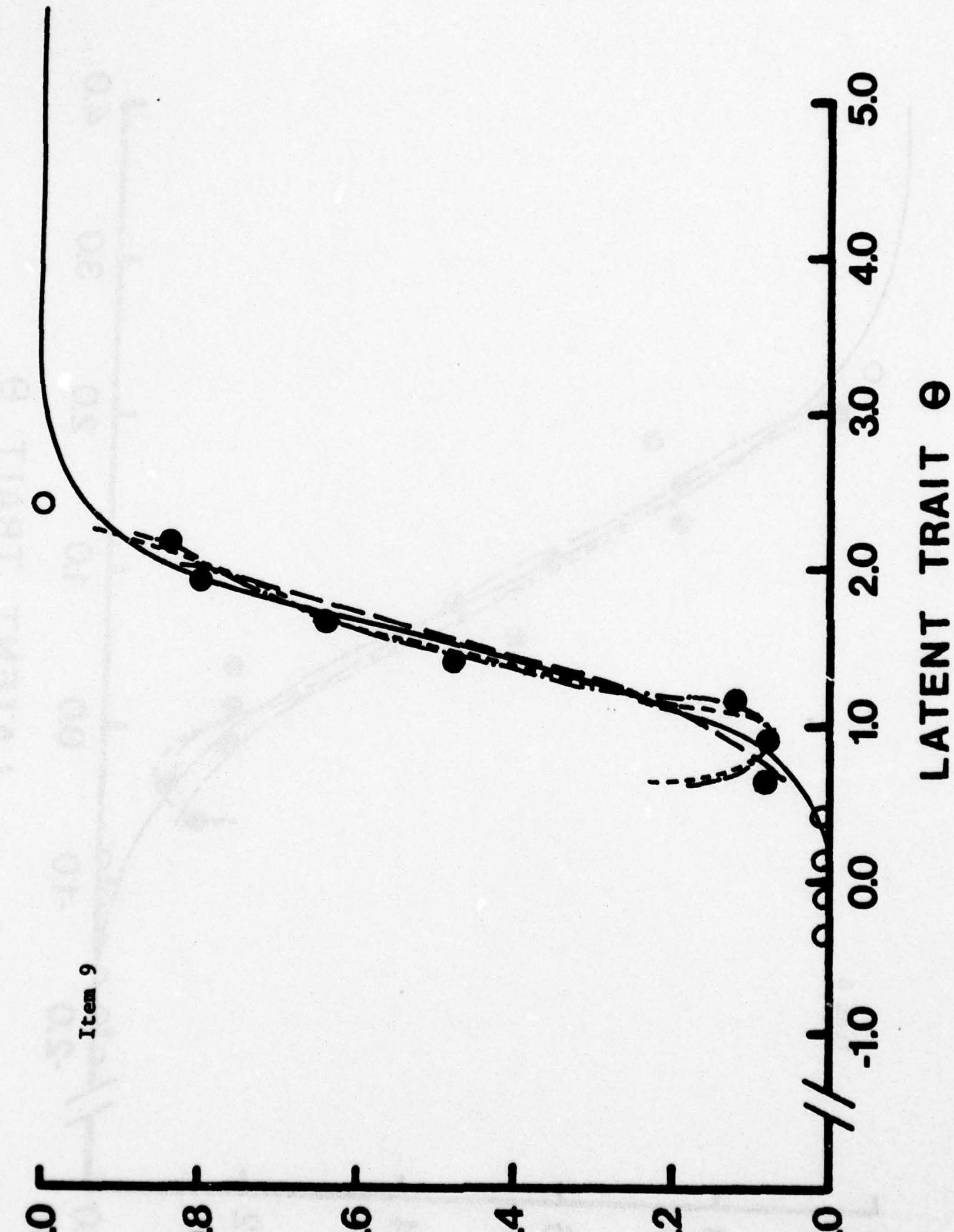

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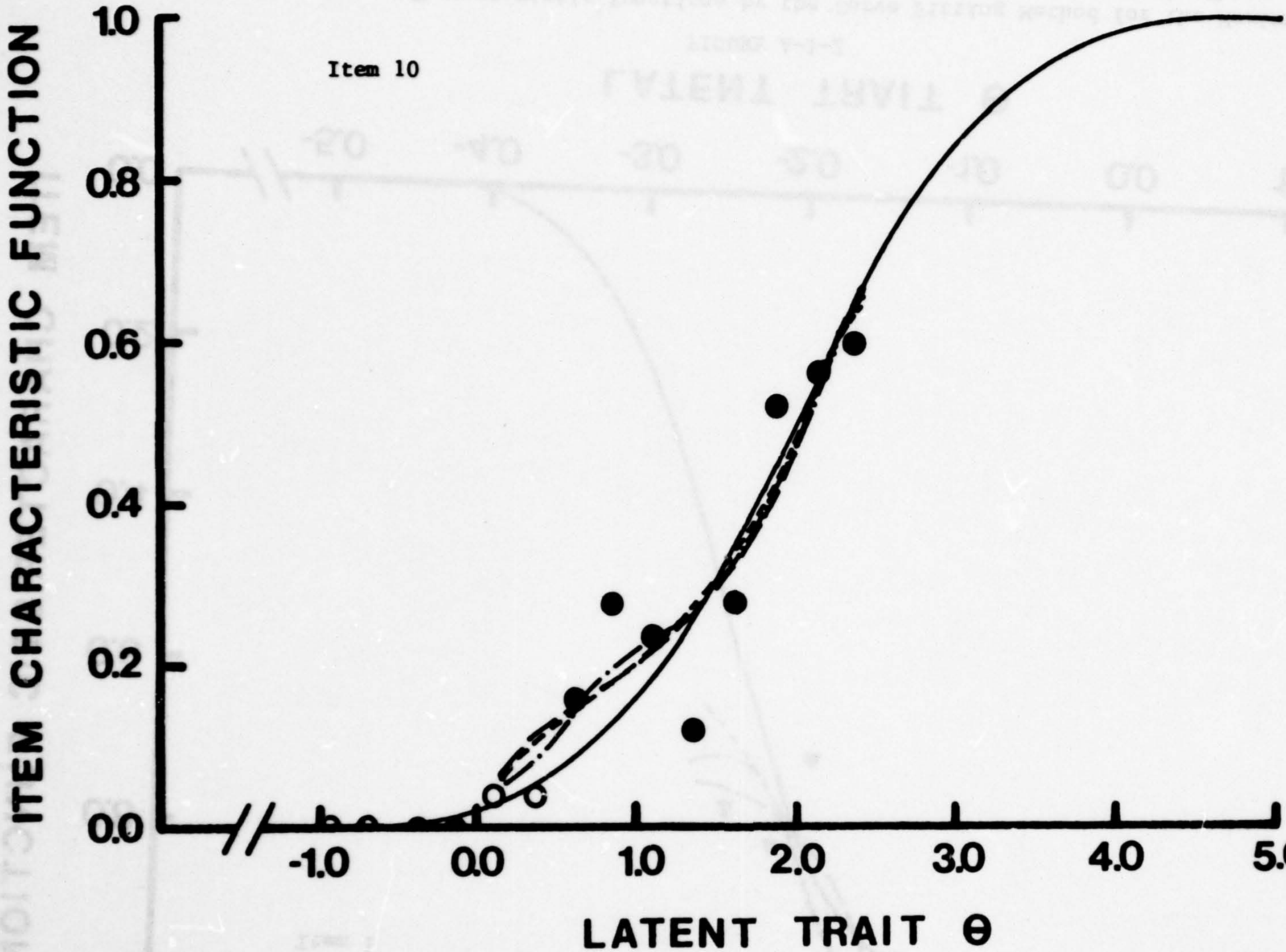

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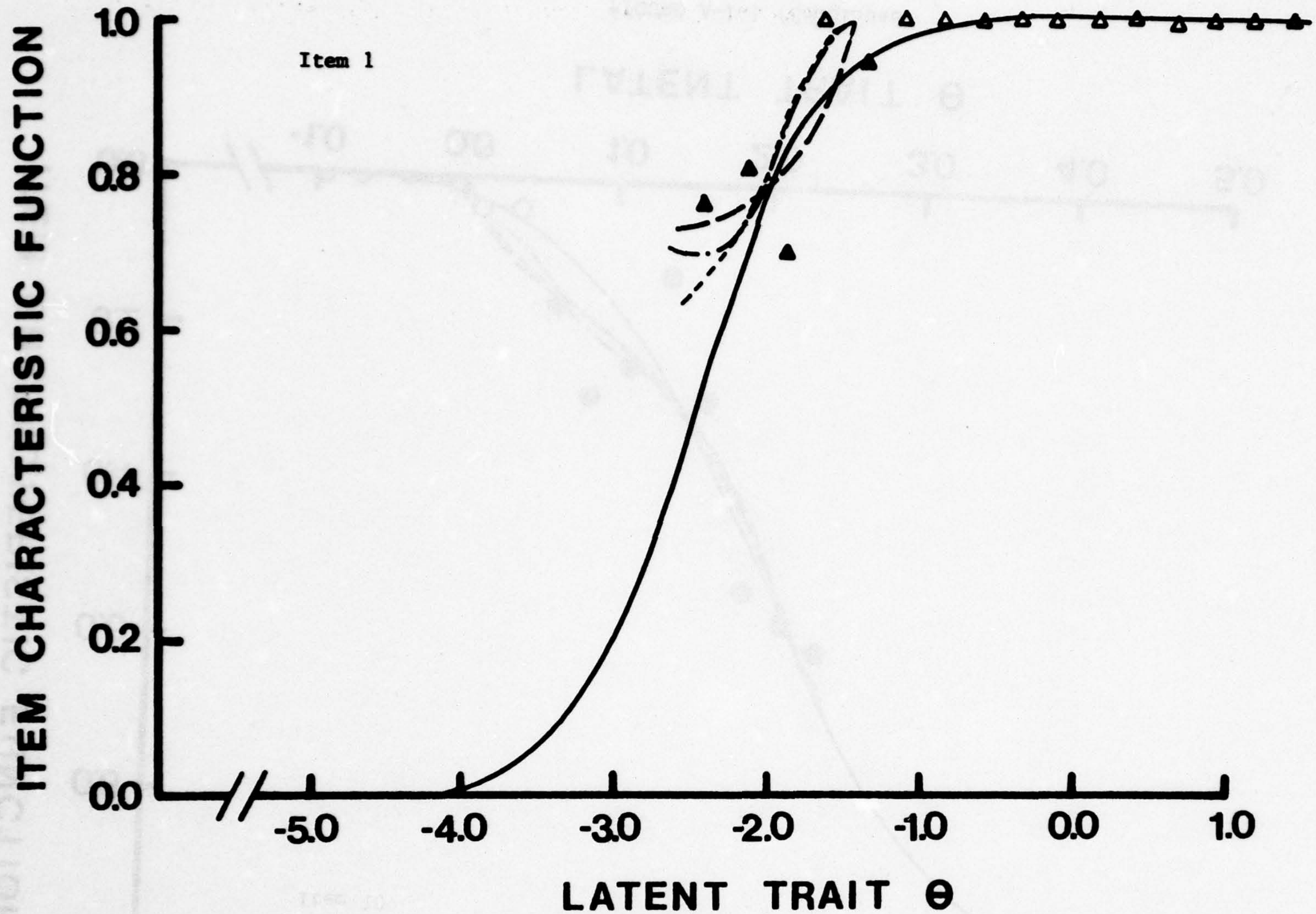
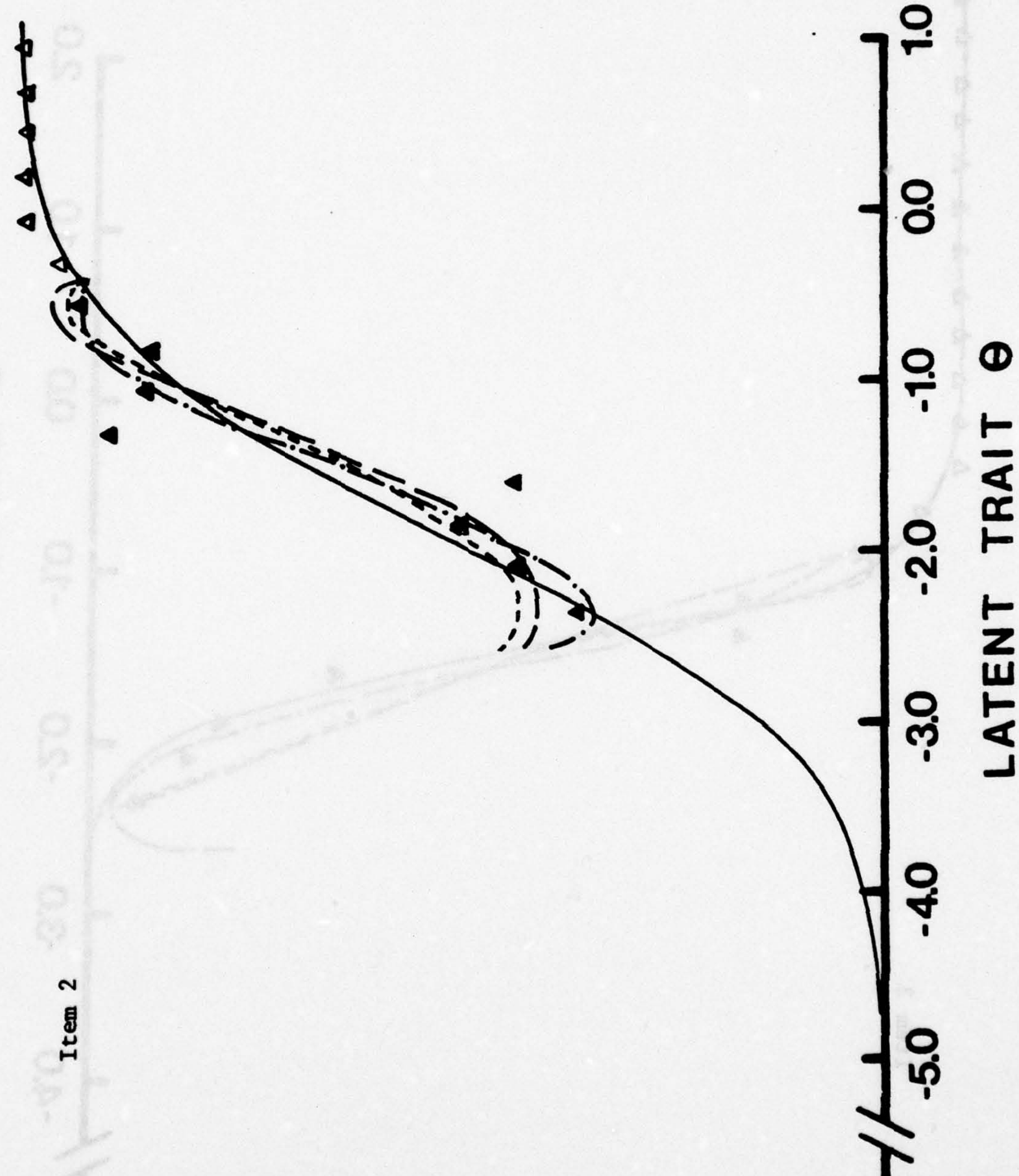


FIGURE A-1-2

Estimated Item Characteristic Functions by the Curve Fitting Method for the Maximum Likelihood Estimate $\hat{\theta}$, Using the Polynomials of Degree 3 (Broken Curve), Those of Degree 4 (Dashed Curve) and Those of Degree 5 (Broken and Dotted Curve), with the Frequency Ratio of $\hat{\theta}$ (Triangles) and the True Item Characteristic Function (Solid Curve)



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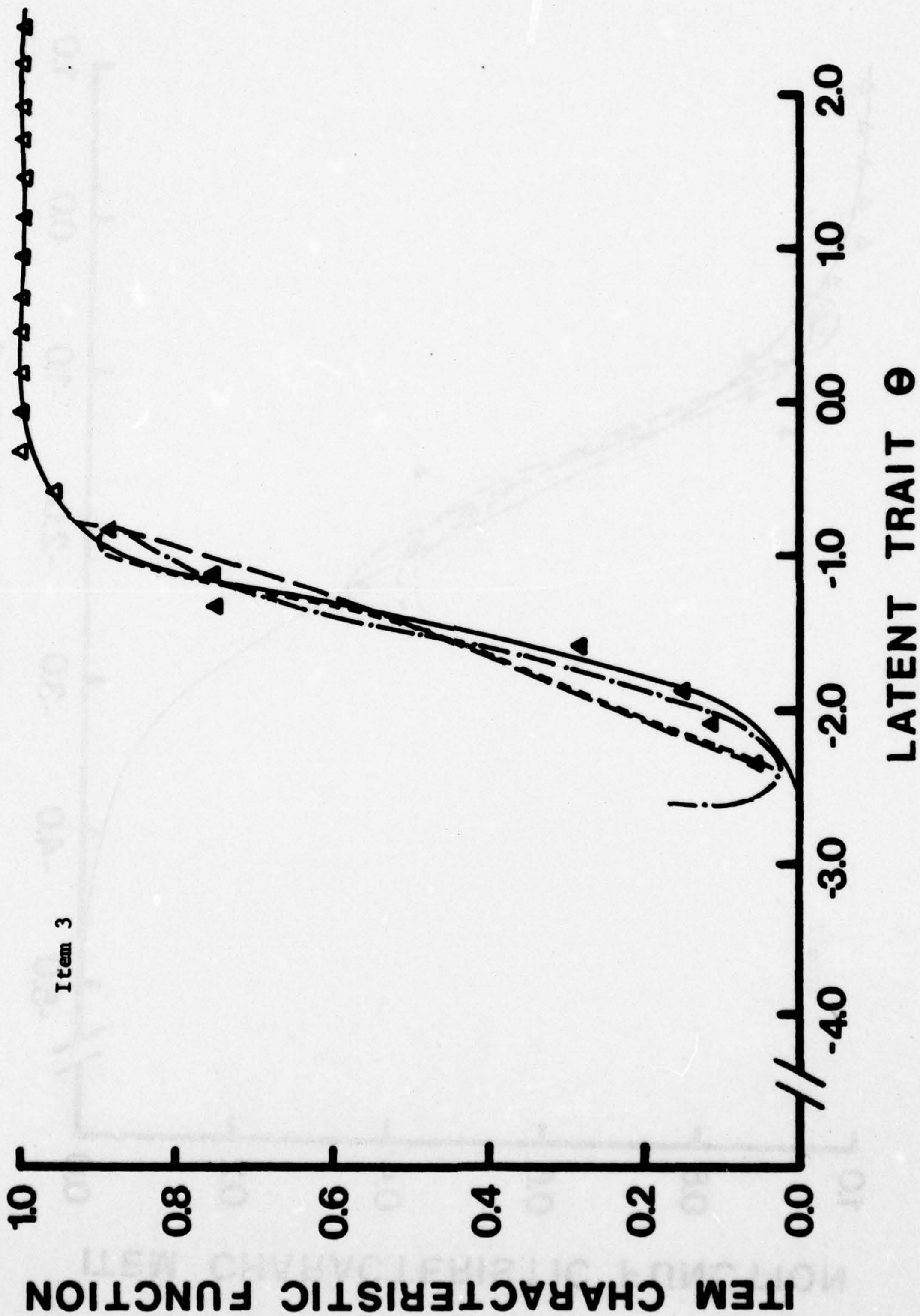

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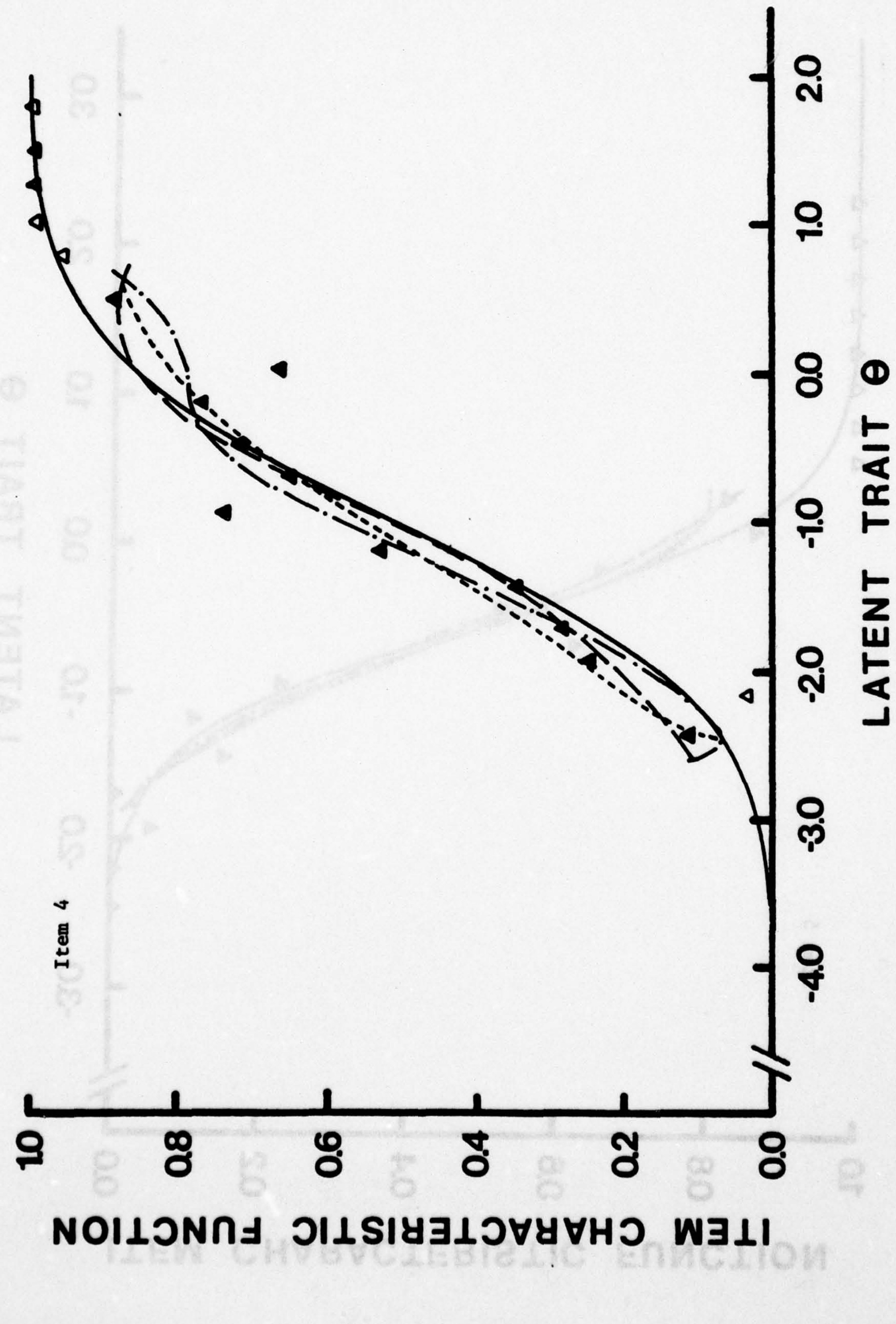

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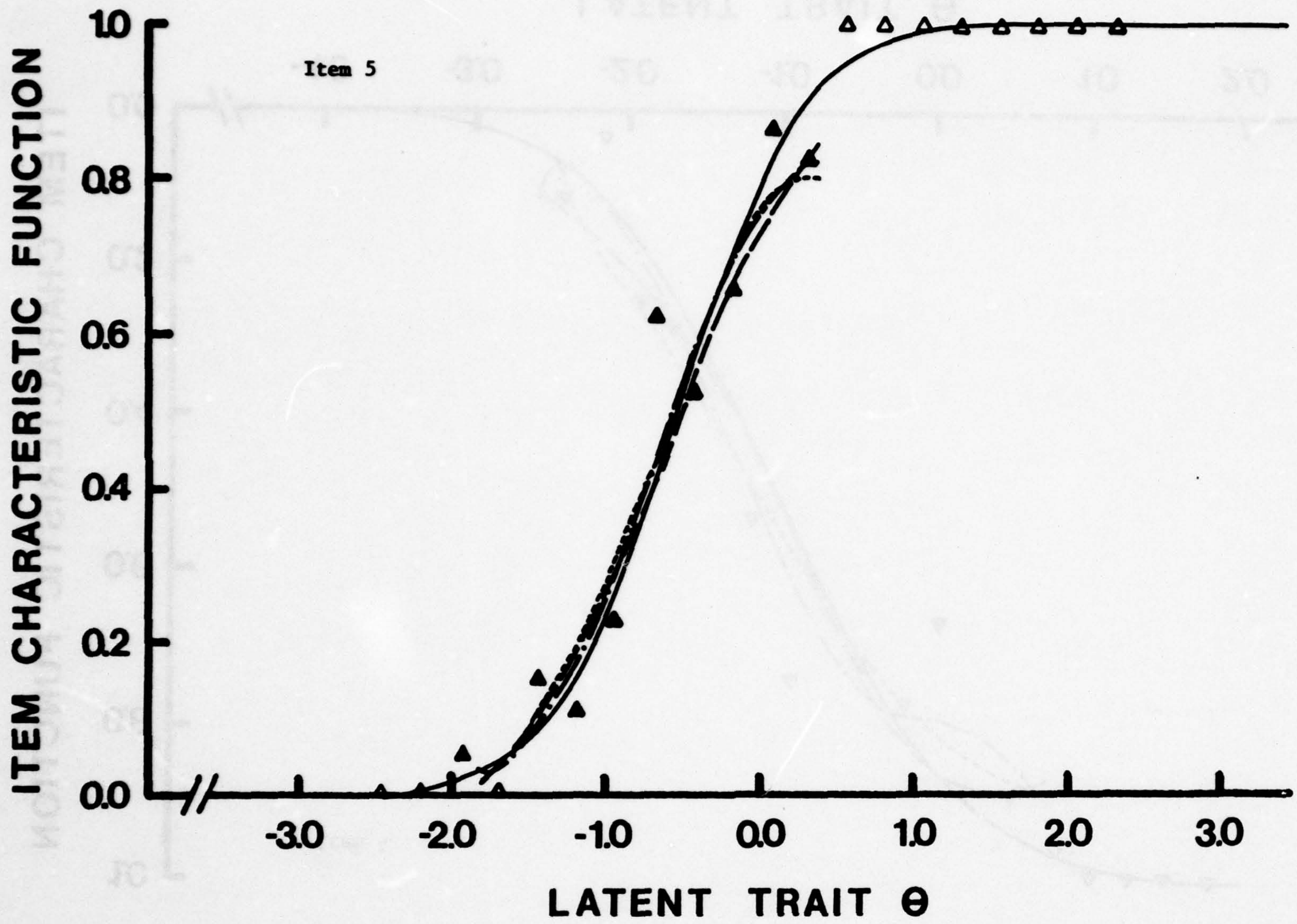

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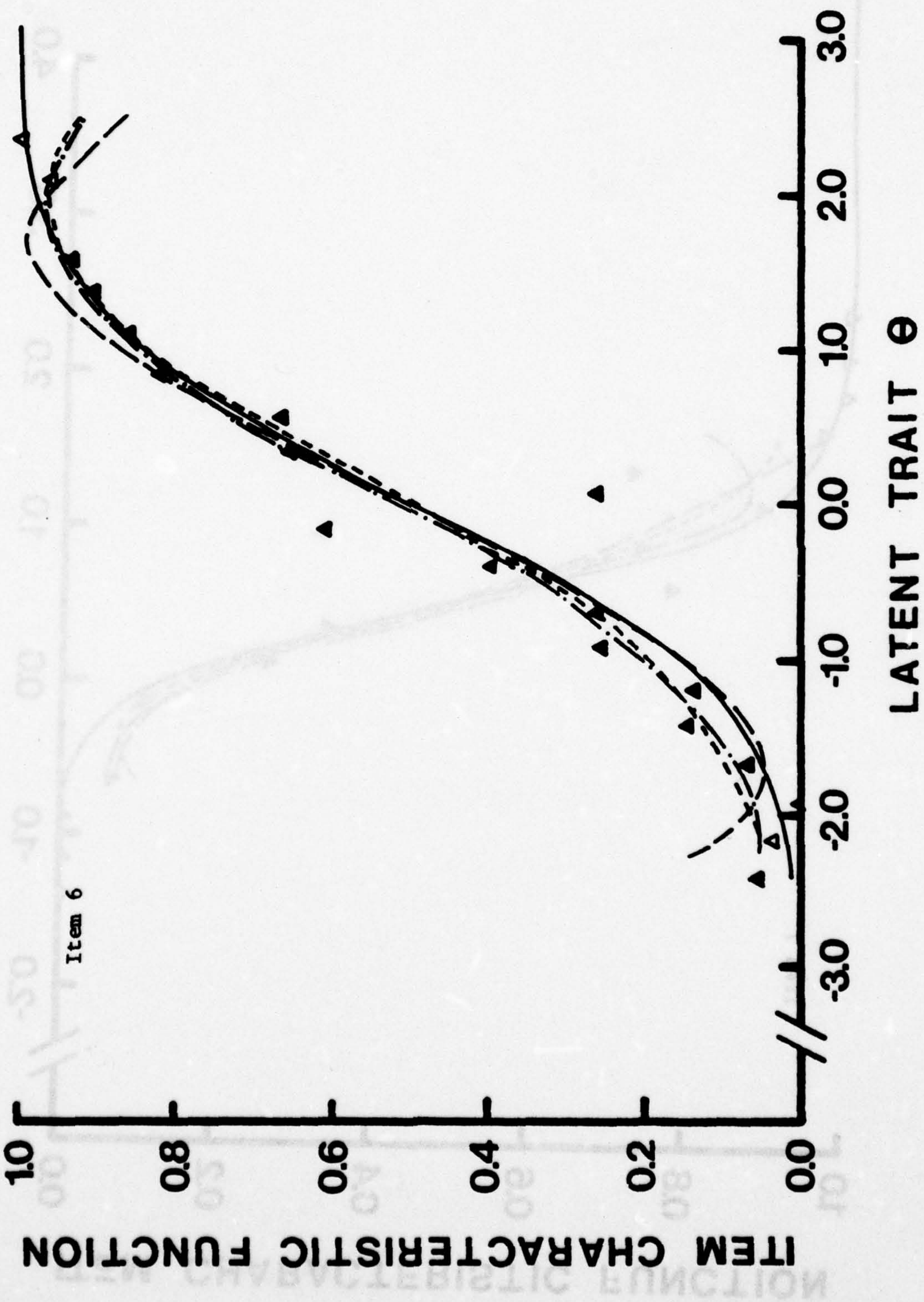

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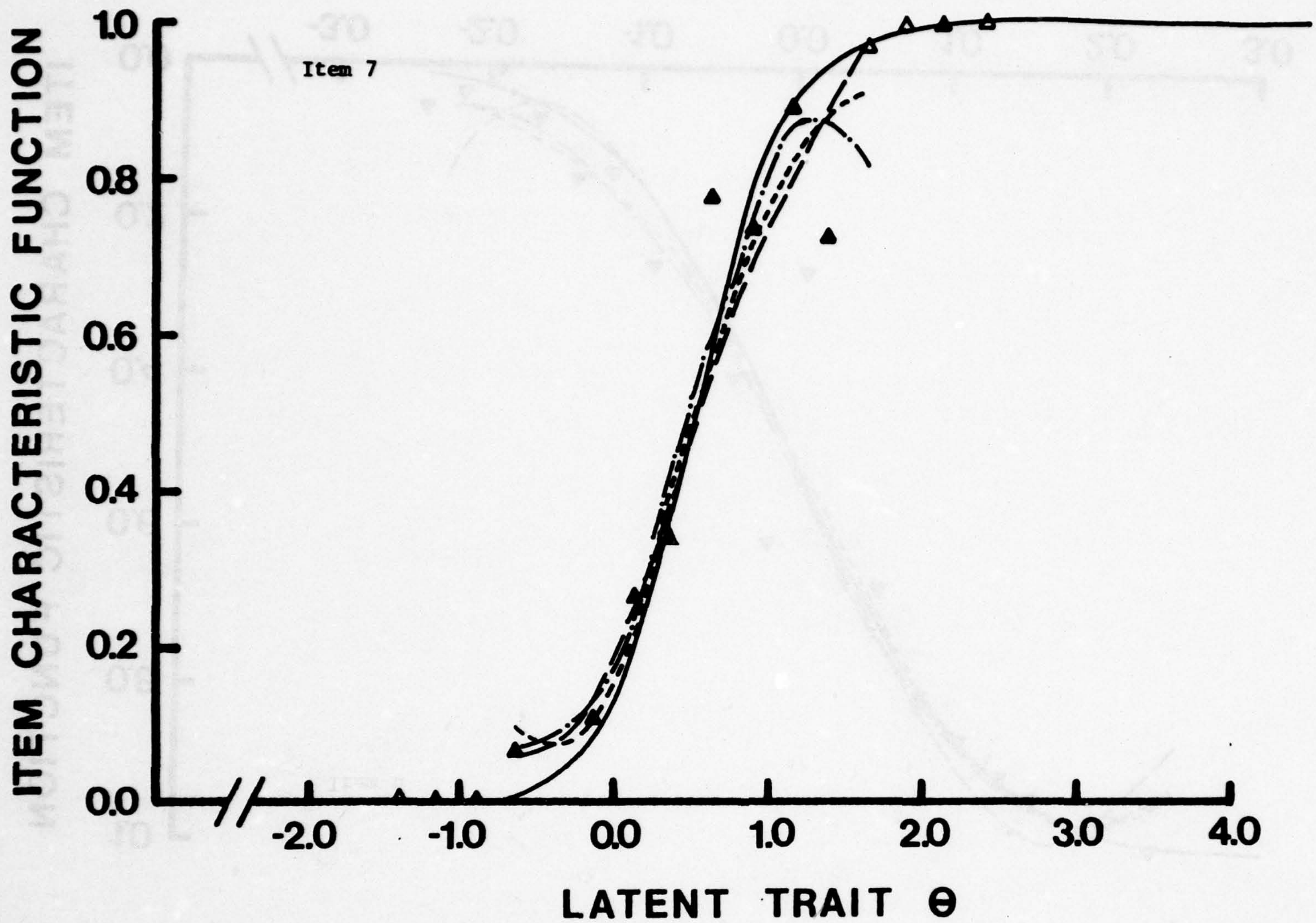

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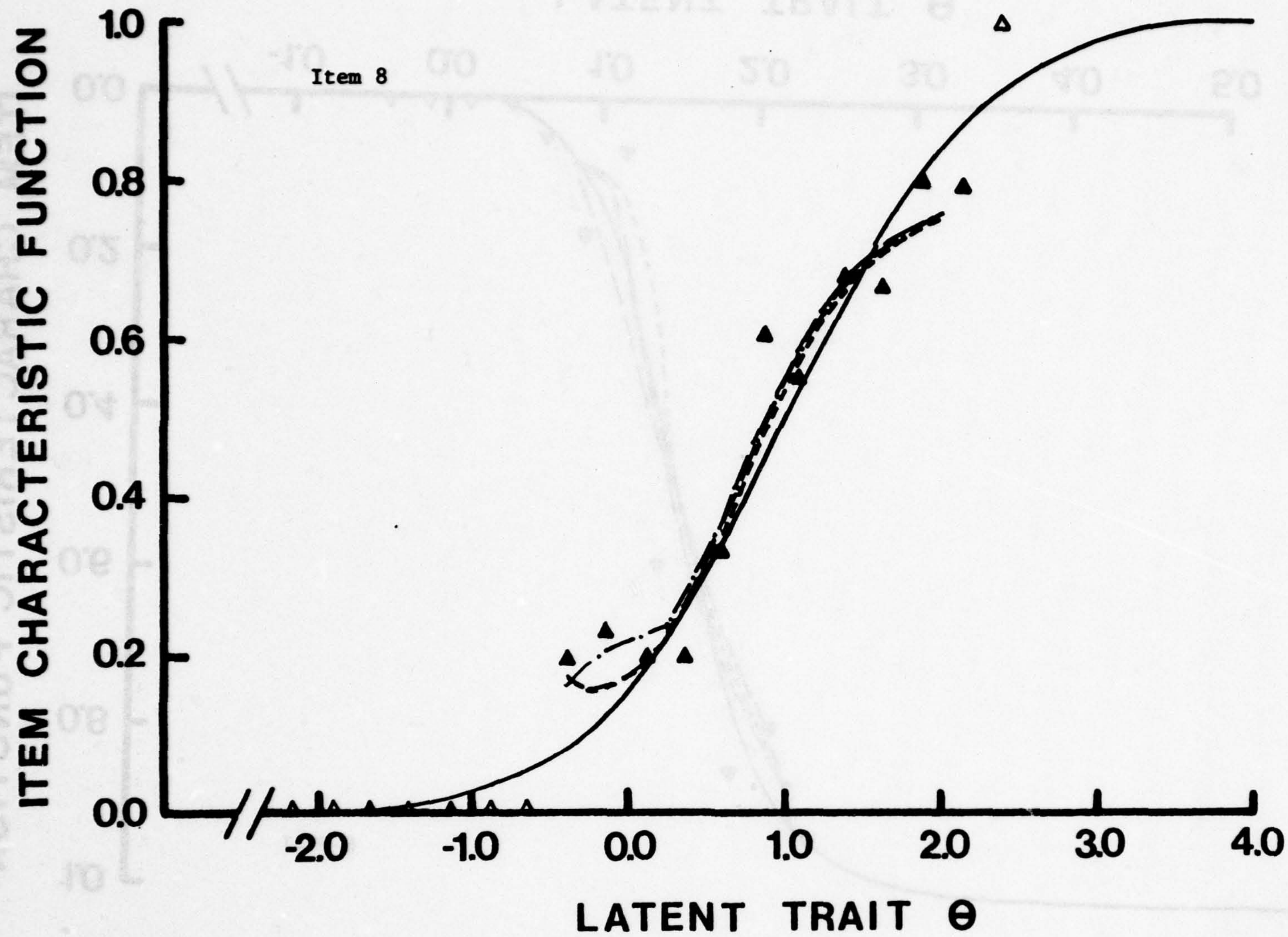

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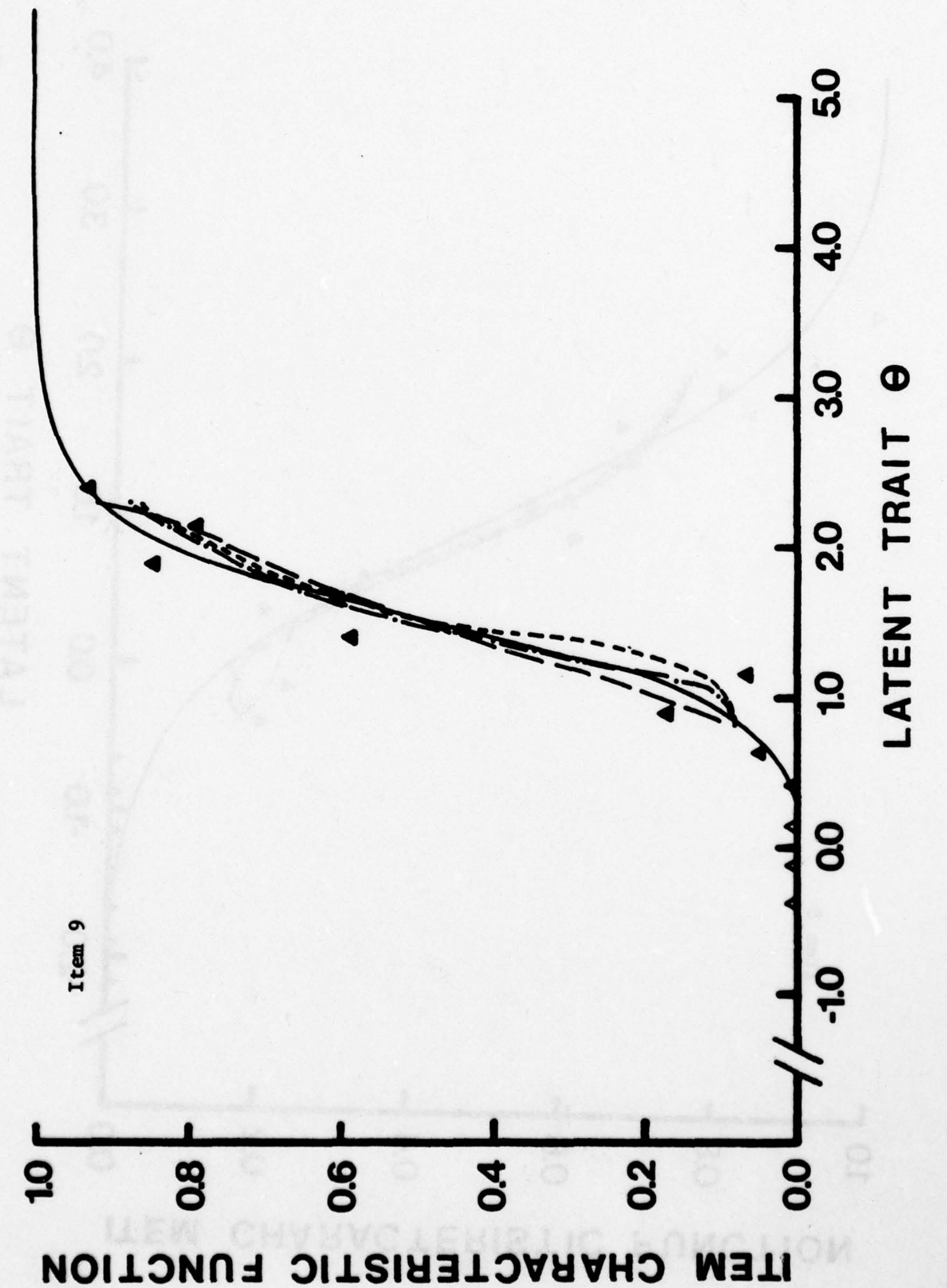

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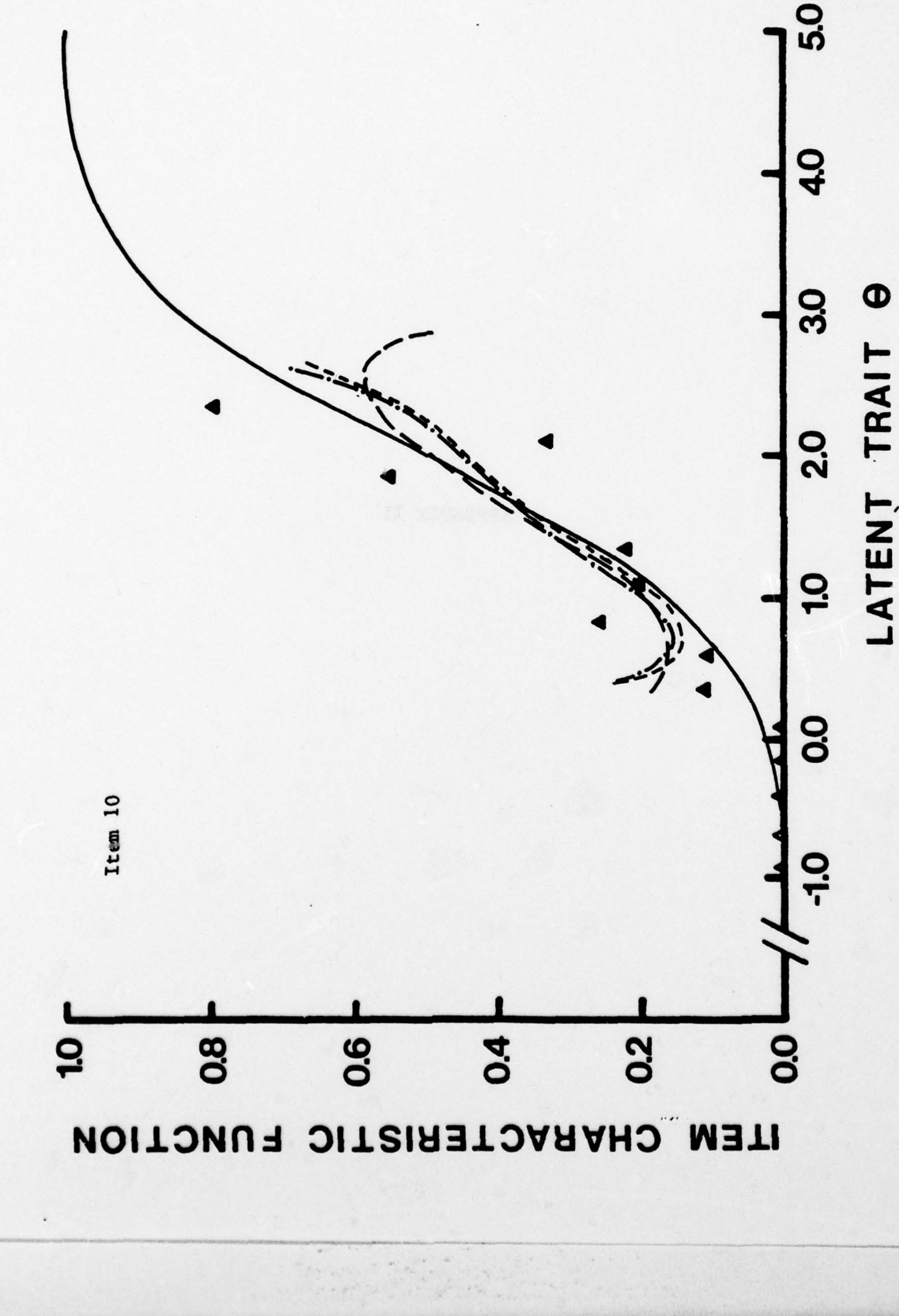

FIGURE A-1-2 (Continued)

APPENDIX II

TABLE A-2-1

Discrimination Parameter and Its Estimates of the Ten Binary Items Obtained by the Conditional P.D.F. Method for $\tilde{\theta}$ of the [illegible]-Parameter Beta Method, with the Estimate Obtained from the Criterion Item Characteristic Function, Using Each of the Four Different Ranges of the Estimated Item Characteristic Functions, [0.15, 0.85], [0.10, 0.90], [0.05, 0.95] and [0.01, 0.99], within the Interval of θ, [-4.0, 4.0].

METHOD		DGR. 3				DGR. 4				CRITERION			
ITEM	TRUE a_g	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99
1	1.5	1.3745	1.3032	1.3228	1.4609	1.3906	1.2660	1.2891	1.4508	0.9536_3	1.1064_4	1.4003_5	1.5517
2	1.0	-0.0634	0.2484	0.3487	0.5126	0.2685	0.4305	0.6083	0.7298	0.8595	0.9693	1.0238	1.0583
3	2.5	1.9931	1.9455	1.3135	1.4843	1.9981	1.0206	1.2222	1.3263	1.9510	1.8923	1.7876	1.8960
4	1.0	0.7770	0.7772	0.7866	0.8087	0.7666	0.7813	0.8122	0.8634	0.8181	0.8114	0.8679	0.8952
5	1.5	1.3052	1.3639	1.3940	1.4538	1.2835	1.3469	1.3785	1.4407	1.3093	1.3464	1.3682	1.4376
6	1.0	0.8715	0.8726	0.9061	0.7958	0.4537	0.5054	0.6024	0.7171	0.8415	0.8585	0.8950	0.8631
7	2.0	1.5721	1.5321	1.4886	1.4937	1.5690	1.5257	1.4795	1.4835	1.5710	1.5214	1.4725	1.4814
8	1.0	0.8344	0.8502	0.9112	1.0319	0.8338	0.8525	0.9193	1.0362	0.8248	0.8460	0.8860	0.9906
9	2.0	1.8703	1.7723	1.6760	1.7023	1.8926	1.7907	1.7093	1.7238	1.8275_5	1.7542	1.7165	1.7445
10	1.0	0.5775	0.5930	0.6592	0.4900	0.6640	0.6690	0.7243	0.7963	0.7128	0.7076	0.7255	0.8839

The number of intervals used in estimation is shown as a subscript when it is less than 6 .

TABLE A-2-2

Difficulty Parameter and Its Estimates of the Ten Binary Items Obtained by the Conditional P.D.F. Method for $\tilde{\theta}$ of the Two-Parameter Beta Method, with the Estimate Obtained from the Criterion Item Characteristic Function, Using Each of the Four Different Ranges of the Estimated Item Characteristic Functions, [0.15, 0.85], [0.10, 0.90], [0.05, 0.95] and [0.01, 0.99], within the Interval of θ, [-4.0, 4.0].

METHOD		DGR. 3				DGR. 4				CRITERION			
ITEM	TRUE b_g	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99
1	-2.5	-2.6889	-2.6828	-2.6836	-2.6801	-2.7333	-2.7399	-2.7374	-2.7125	-2.8510_3	-2.7703_4	-2.6506_5	-2.6001
2	-2.0	2.0827	-3.6770	-3.3098	-2.9874	-2.6406	-2.5500	-2.4785	-2.4267	-2.0081	-2.0072	-2.0022	-1.9943
3	-1.5	-1.4779	-1.4733	-1.5272	-1.6189	-1.4870	-1.5505	-1.5525	-1.6085	-1.4896	-1.4945	-1.5068	-1.5252
4	-1.0	-1.0915	-1.0572	-1.0713	-1.0930	-1.1033	-1.0625	-1.0559	-1.0423	-1.0589	-1.0572	-1.0048	-1.0148
5	-0.5	-0.4881	-0.4698	-0.4624	-0.4748	-0.4945	-0.4744	-0.4659	-0.4755	-0.4885	-0.4732	-0.4719	-0.4817
6	0.0	-0.0474	-0.0638	-0.0739	-0.0012	0.1424	0.1138	0.0473	0.0904	-0.0531	-0.0640	-0.0747	-0.0354
7	0.5	0.4970	0.5181	0.5231	0.5137	0.5030	0.5237	0.5282	0.5177	0.5000	0.5251	0.5268	0.5191
8	1.0	0.9675	0.9601	0.9584	0.9515	0.9733	0.9618	0.9532	0.9459	0.9745	0.9645	0.9812	0.9779
9	1.5	1.4945	1.5020	1.5053	1.5048	1.5018	1.5067	1.5061	1.5071	1.4963_5	1.5088	1.5016	1.5064
10	2.0	2.2516	2.2555	2.2594	2.8357	2.1708	2.1716	2.1705	2.1611	2.1237	2.1256	2.1176	2.0408

The number of intervals used in estimation is shown as a subscript when it is less than 6 .

TABLE A-2-3

Discrimination Parameter and Its Estimates of the Ten Binary Items Obtained by the Conditional P.D.F. Method for $\tilde{\theta}$ of the Two-Parameter Beta Method, with the Estimate Obtained from the Criterion Item Characteristic Function, Using Each of the Four Different Ranges of the Estimated Item Characteristic Functions, [0.15, 0.85], [0.10, 0.90], [0.05, 0.95] and [0.01, 0.99], within the Interval of θ, [-2.4, 2.4].

ITEM \ METHOD	TRUE a_g	DGR. 3				DGR. 4				CRITERION			
		0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99
1	1.5	0.717_3	0.844_4	1.119_5	1.543	0.626_3	0.804_4	1.112_5	1.541	0.954_3	1.106_4	1.400_5	1.552
2	1.0	0.879	1.039	1.051	1.082	0.921	1.032	1.067	1.091	0.859	0.969	1.024	1.058
3	2.5	1.993	1.945	1.785	1.900	1.998	1.998	1.777	1.828	1.951	1.892	1.788	1.896
4	1.0	0.777	0.872	0.887	0.907	0.767	0.866	0.875	0.895	0.818	0.811	0.868	0.895
5	1.5	1.305	1.364	1.394	1.454	1.284	1.347	1.378	1.441	1.309	1.346	1.368	1.438
6	1.0	0.871	0.873	0.906	0.863	0.863	0.863	0.897	0.857	0.841	0.859	0.895	0.863
7	2.0	1.572	1.532	1.489	1.494	1.569	1.526	1.480	1.484	1.571	1.521	1.473	1.481
8	1.0	0.834	0.850	0.911	0.979	0.834	0.852	0.919	0.983	0.825	0.846	0.886	0.991
9	2.0	1.870	1.772	1.676	1.691	1.893	1.791	1.709	1.720	1.827_5	1.754	1.716	1.745
10	1.0	0.686	0.686	0.762	0.862	0.695	0.695	0.769	0.869	0.713	0.708	0.725	0.884

The number of intervals used in estimation is shown as a subscript when it is less than 6.

TABLE A-2-4

Difficulty Parameter and Its Estimates of the Ten Binary Items Obtained by the Conditional P.D.F. Method for $\tilde{\theta}$ of the Two-Parameter Beta Method, with the Estimate Obtained from the Criterion Item Characteristic Function, Using Each of the Four Different Ranges of the Estimated Item Characteristic Functions, [0.15, 0.85], [0.10, 0.90], [0.05, 0.95] and [0.01, 0.99], within the Interval of θ, [-2.4, 2.4].

ITEM \ METHOD	TRUE b_g	DGR. 3 0.15-0.85	DGR. 3 0.10-0.90	DGR. 3 0.05-0.95	DGR. 3 0.01-0.99	DGR. 4 0.15-0.85	DGR. 4 0.10-0.90	DGR. 4 0.05-0.95	DGR. 4 0.01-0.99	CRITERION 0.15-0.85	CRITERION 0.10-0.90	CRITERION 0.05-0.95	CRITERION 0.01-0.99
1	-2.5	-3.148_3	-3.016_4	-2.815_5	-2.624	-3.312_3	-3.081_4	-2.837_5	-2.637	-2.851_3	-2.770_4	-2.651_5	-2.600
2	-2.0	-1.967	-1.970	-1.968	-1.962	-1.963	-1.967	-1.965	-1.961	-2.008	-2.007	-2.002	-1.994
3	-1.5	-1.478	-1.473	-1.494	-1.514	-1.487	-1.487	-1.505	-1.515	-1.490	-1.494	-1.507	-1.525
4	-1.0	-1.091	-1.006	-0.992	-1.000	-1.103	-1.012	-1.001	-1.009	-1.059	-1.057	-1.005	-1.015
5	-0.5	-0.488	-0.470	-0.462	-0.475	-0.495	-0.474	-0.466	-0.475	-0.489	-0.473	-0.472	-0.482
6	0.0	-0.047	-0.064	-0.074	-0.042	-0.045	-0.063	-0.075	-0.039	-0.053	-0.064	-0.075	-0.035
7	0.5	0.497	0.518	0.523	0.514	0.503	0.524	0.528	0.518	0.500	0.525	0.527	0.519
8	1.0	0.968	0.960	0.958	0.989	0.973	0.962	0.953	0.982	0.974	0.965	0.981	0.978
9	1.5	1.494	1.502	1.505	1.508	1.502	1.507	1.506	1.508	1.496_5	1.509	1.502	1.506
10	2.0	2.147	2.147	2.108	2.057	2.142	2.142	2.104	2.054	2.124	2.126	2.118	2.041

The number of intervals used in estimation is shown as a subscript when it is less than 6.

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